Solutions of Archive - JEE Main & Advanced

Quadratic Equations	Class - XI | Mathematics

JEE Main 2021

1.(C) $p+q=2$

$p^4+q^4=272$

$(p^2+q^2)^2-2p^2q^2=272$

$\left[(p+q)^2-2pq\right]^2-2p^2q^2=272$

$\left((2)^2-2pq\right)^2-2p^2q^2=272$

$\left(4-2pq\right)^2-2p^2q^2=272$

$16+4p^2q^2-16pq-2p^2q^2=272$

$16-16pq+2p^2q^2=272$

$2p^2q^2-16pq-256=0$; $(pq)^2-8pq-128=0$

$pq=\dfrac{8\pm\sqrt{64+512}}{2}=\dfrac{8\pm24}{2}$; $pq=16,\ -8$

2.(2) Case I : $x\geq 5$

$(x+1)^2+x-5=\dfrac{27}{4}$ $\Rightarrow$ $x^2+2x+1+x-5-\dfrac{27}{4}=0$ $\Rightarrow$ $x^2+3x-\dfrac{43}{4}=0$

$D=9+43=52$

Roots $\dfrac{-3+2\sqrt{13}}{2}$ ignore, $\dfrac{-3-2\sqrt{13}}{2}$ ignore

Case II $x<5$

$(x+1)^2+(5-x)=\dfrac{27}{4}$ $\Rightarrow$ $x^2+2x+1+5-x-\dfrac{27}{4}=0$ $\Rightarrow$ $x^2+x-\dfrac{3}{4}=0$ $\Rightarrow 4x^2+4x-3=0$

$\Rightarrow$ $4x^2+6x-2x-3=0$ $\Rightarrow$ $2x(2x+3)-(2x+3)=0$ $\Rightarrow$ $(2x-1)(2x+3)=0$

$\Rightarrow$ $x=\dfrac{1}{2}$ or $x=-\dfrac{3}{2}$

Both acceptable

3.(B) $D<0$

$4\left(3k-1\right)^2-4\left(8k^2-7\right)<0$ $\Rightarrow$ $\left(3k-1\right)^2-8k^2+7<0$ $\Rightarrow$ $9k^2+6k-8k^2+7<0$

$\Rightarrow$ $k^2-6k+8<0$ $\Rightarrow$ $\left(k-4\right)\left(k-2\right)<0$ $\Rightarrow$ $2<k<4$ $\Rightarrow$ $k=3$

4.(C) $\dfrac{a_{10}-2a_8}{3a_9}=\dfrac{\left(\alpha^{10}-\beta^{10}\right)-2\left(\alpha^8-\beta^8\right)}{3\left(\alpha^9-\beta^9\right)}=\dfrac{\alpha^8\left(\alpha^2-2\right)-\beta^8\left(\beta^2-2\right)}{3\left(\alpha^9-\beta^9\right)}$ .... (A)

Also $\alpha^2 - 6\alpha - 2 = 0 \Rightarrow \alpha^2 - 2 = 6\alpha$

Similarly $\beta^2 - 2 = 6\beta$

$\therefore$ from (A)

$$\frac{a_{10} - 2a_8}{3a_9} = \frac{\alpha^9.6\alpha - \beta^8.6\beta}{3\left(\alpha^9 - \beta^9\right)} = \frac{6\left(\alpha^9 - \beta^9\right)}{3\left(\alpha^9 - \beta^9\right)} = 2$$

5.(324) $\alpha + \beta = 1 \qquad \alpha\beta = -1$

$p_n = \alpha^n + \beta^n$

$p_{n-1} = 11 \qquad p_{n+1} = 29$

$p_n^2 = ?$

Equation whose roots are α, β is

$x^2 - x - 1 = 0.$

$\alpha^2 - \alpha - 1 = 0$

$\alpha^2 = \alpha + 1$

$\alpha^n = \alpha^{n-1} + \alpha^{n-2}$

$\beta^n = \beta^{n-1} + \beta^{n-2}$

$\alpha^n + \beta^n = \left(\alpha^{n-1} + \beta^{n-1}\right) + \left(\alpha^{n-2} + \beta^{n-2}\right)$

$p_n = p_{n-1} + p_{n-2}$

$p_{n+1} = p_n + p_{n-1} \qquad \Rightarrow \qquad p_n = p_{n+1} - p_{n-1}$

$p_n = 29 - 11 = 18$

$p_n^2 = 324$

6.(5) $f : [-1,1] \to R \qquad f(x) = ax^2 + bx + c$

$f(-1) = 2 \Rightarrow a - b + c = 2 \qquad \ldots(i)$

$f'(-1) = 1 \qquad -2a + b = 1 \qquad \ldots(ii)$

Also, $f''(x) = 2a = \frac{1}{2} \Rightarrow a = \frac{1}{4}$

$b = 1 + \frac{1}{2} = \frac{3}{2}$

$c = 2 - \frac{1}{4} + \frac{3}{2} = \frac{8 - 1 + 6}{4} = \frac{13}{4}$

$f(x) = \frac{1}{4}x^2 + \frac{3}{2}x + \frac{13}{4} = \frac{1}{4}(x^2 + 6x + 13)$

$f'(x) = \frac{1}{4}(2x + 6) > 0, \; x \in [-1,1]$

So, $f(1) = \frac{1}{4}(1 + 6 + 13) = 5$

$\Rightarrow \quad f(x) \le 5$

$\alpha_{\min} = 5$

7.(D) Assuming the continued fraction is convergent

$$x = 3 + \frac{1}{4 + \frac{1}{x}} \Rightarrow x = 3 + \frac{1}{\frac{4x + 1}{x}}$$

$$\Rightarrow \qquad x = 3 + \frac{x}{4x + 1}$$

$$\Rightarrow \qquad 4x^2 + x = 12x + 3 + x$$

$\Rightarrow \quad 4x^2 - 12x - 3 = 0 \quad \Rightarrow \quad x = \frac{12 + \sqrt{144 + 48}}{8}$ (Rejecting the negative value)

$x = \frac{3 + \sqrt{9 + 3}}{2} = \frac{3 + \sqrt{12}}{2}$

8.(B) Let $4 + \frac{1}{5 + \frac{1}{y}}$

$y = 4 + \frac{y}{5y + 1}$

$(5y + 1)y = 20y + 4 + y$

$5y^2 + y = 21y + 4$

$5y^2 - 20y - 4 = 0$

$y = \frac{70 \pm \sqrt{400 + 80}}{10} = \frac{20 \pm 4\sqrt{30}}{10} = 2 \pm \frac{2}{5}\sqrt{30}$

$y > 0 \Rightarrow y = 2 + \frac{2}{5}\sqrt{30}$

9.(B) $\because \quad x^2 + \sqrt{3} = -3^{\frac{1}{4}}x \quad \Rightarrow \quad x^4 + 2\sqrt{3}x^2 + 3 = \sqrt{3}x^2$

$\Rightarrow \quad x^4 + 3 = -\sqrt{3}x^2 \Rightarrow \quad x^8 + 6x^4 + 9 = 3x^4 \quad \Rightarrow \quad x^8 + 3x^4 + 9 = 0$

$\because \quad \alpha$ and β are the roots of the above equation $\quad \therefore \quad \alpha^8 = -9 - 3\alpha^4$

$\Rightarrow \quad \alpha^{12} = -9\alpha^4 - 3\alpha^8 = -9\alpha^4 - 3\left(-9 - 3\alpha^4\right) \quad \Rightarrow \quad \alpha^{12} = 27$

Similarly $\beta^{12} = 27$

$\therefore \quad \alpha^{96}\left(\alpha^{12} - 1\right) + \beta^{96}\left(\beta^{12} - 1\right) = (27)^8 \times 26 + (27)^8 \times 26 = 52 \times 27^8 = 52 \times 3^{24}$

10.(1) $\log_{x+1}(x+1)(2x+5) + \log_{2x+5}(x+1)^2 = 4$

$1 + \frac{\log(2x+5)}{\log(x+1)} + 2\frac{\log(x+1)}{\log(2x+5)} = 4$

$\lambda + \frac{2}{\lambda} = 3; \quad \lambda = 1, 2$

Case 1 $\lambda = 1$

$\log(2x+5) = \log(x+1)$

$2x + 5 = x + 1 \quad x = -4$

Case 2 $\lambda = 2$

$\log(2x+5) = \log(x+1)^2$

$2x + 5 = x^2 + 2x + 1$

$x^2 = 5 - 1$

$x = \pm 2; \; x = 2$

11.(1) α and β are the roots of $x^2 + 5\sqrt{2}x + 10 = 0$

$P_n = \alpha^n - \beta^n$

$\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2}$

$$= \frac{P_{17}\left(P_{20}+5\sqrt{2}P_{19}\right)}{P_{18}\left(P_{19}+5\sqrt{2}P_{18}\right)} = \frac{P_{17}}{P_{18}}\left[\frac{\alpha^{20}-\beta^{20}+5\sqrt{2}\left(\alpha^{19}-\beta^{19}\right)}{\alpha^{19}-\beta^{19}+5\sqrt{2}\left(\alpha^{18}-\beta^{18}\right)}\right] = \frac{P_{17}}{P_{18}}\left[\frac{\alpha^{19}\left(\alpha+5\sqrt{2}\right)-\beta^{19}\left(\beta+5\sqrt{2}\right)}{\alpha^{18}\left(\alpha+5\sqrt{2}\right)-\beta^{18}\left(\beta+5\sqrt{2}\right)}\right]$$

$\alpha^2+5\sqrt{2}\alpha+10=0 \Rightarrow \alpha\left(\alpha+5\sqrt{2}\right)=-10 \Rightarrow \alpha+5\sqrt{2}=-10/\alpha$

$\beta^2+5\sqrt{2}\beta+10=0 \quad \beta+5\sqrt{2}=-10/\beta = \frac{P_{17}}{P_{18}}\left[\frac{\alpha^{19}.\left(-10/\alpha\right)-\beta^{19}\left(-10/\beta\right)}{\alpha^{18}\left(-10/\alpha\right)-\beta^{18}\left(-10/\beta\right)}\right] = \frac{P_{17}}{P_{18}}\times\frac{P_{18}}{P_{17}}=1$

12.(B) say $|x| = t$

$t^2 - t - 12 = 0$

$(t-4)(t+3) = 0$

$\Rightarrow$ $t = 4$ or $t = -3$ $\Rightarrow$ $|x|$ can't be negative $\Rightarrow$ $|x| = 4$ $\Rightarrow$ $x = \pm 4$

13.(13) a, b, c are the roots of the equation

$x^3 - x^2 + 2x - 3 = 0$

$a^2+b^2+c^2 = (a+b+c)^2 - 2(ab+bc+ca) = 1-4 = -3$

$a^3+b^3+c^3 = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)+3abc = (-3-2)+9 = 4$

$x^4 = x^3 - 2x^2 + 3x$

$a^4+b^4+c^4 = \Sigma a^3 - 2\,\Sigma a^2 + 3\,\Sigma a = 4 - 2(-3) + 3.1 = 13$

14.(C) $x^2 + (20)^{1/4}\cdot x + 5^{1/2} = 0$

$\alpha \quad \beta$

$x^2+\sqrt{5} = -(20)^{1/4}\cdot x \quad \Rightarrow x^4+5+2\sqrt{5}\cdot x^2 = 2\sqrt{5}x^2 \quad \Rightarrow x^4 = -5 \Rightarrow x^8 = 25$

$\alpha^8+\beta^8 = 25+25 = 50$

15.(2) $e^{2x} - e^x - 4 - \frac{1}{e^x} + \frac{1}{e^{2x}} = 0$

$e^{2x} + \frac{1}{e^{2x}} - \left(e^x + \frac{1}{e^x}\right) - 4 = 0$, $e^x + \frac{1}{e^x} = t$

$t^2 - 2 - t - 4 = 0$

$t^2 - t - 6 = 0$

$t = 3$, $t = -2$ Not possible

$e^x + \frac{1}{e^x} = 3$ 2 value of x .

16.(D)

$x^2 + ax + b = 0 \nearrow \alpha$, $\searrow \beta$

Case – I $\alpha = \beta$

$\alpha = \alpha^2 - 2$

$\alpha^2 - \alpha - 2 = 0$

$\alpha = 2$ or -1

Now $\alpha + \alpha = -a$

$2\alpha = -a$

$a = -2\alpha$

$a = -4$ or $a = 2$

$\alpha\cdot\alpha = b$

$\alpha^2 = b$

$b = 4$ or $b = 1$

$(a, b) = (-4, 4)$ or $(2, 1)$

Case – 2 $\alpha \neq \beta$

(i) $\alpha = \alpha^2 - 2$ and $\beta = \beta^2 - 2$

$\alpha, \beta = 2, -1$

$\alpha + \beta = -a$ and $\alpha\beta = b$

$a = -1$ $b = -2$

$(a, b) \equiv (-1, -2)$

(ii) $\alpha = \beta^2 - 2$ and $\beta = \alpha^2 - 2$

$\alpha - \beta = (\beta - \alpha)(\beta + \alpha)$

$\Rightarrow$ $\beta + \alpha = -1$

$-a = -1$

$a = 1$

$\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta - 4$

$-1 = -3 - 2\alpha\beta$

$\Rightarrow$ $\alpha\beta = -1 = b$

$b = -1$

$(a, b) \equiv (1, -1)$

(iii) $\alpha = \alpha^2 - 2 = \beta^2 - 2$

$\alpha = -\beta$

and $\alpha = 2$ or -1

$\beta = -2$ or 1

$\alpha + \beta = -a \Rightarrow a = 0$

$\alpha\beta = b = -4$ or -1

$(a, b) \equiv (0, -4)$ or $(0, -1)$

$\therefore$ 6 pairs of (a, b)

17.(D) Let $3x^2 + 4x + 2 = t$

$(t+1)^2 - (k+1)(t+1)(t) + k(t^2) = 0$

$(t^2 + 2t + 1) - (k+1)(t^2 + t) + kt^2 = 0$

$2t + 1 - (k+1)t = 0$

$(1-k)(3x^2 + 4x + 2) + 1 = 0$

$3(1-k)x^2 + 4(1-k)x + (3 - 2k) = 0$

$k \neq 1$ $D \geq 0$

$16(1-k)^2 - 4.3(1-k)(3-2k) \geq 0$

$4\left[4(k^2 - 2k + 1) - 3(2k^2 - 5k + 3)\right] \geq 0$

$-2k^2 + 7k - 5 \geq 0$

$(k-1)(2k-5) \leq 0$

$k \in \left(1, \frac{5}{2}\right]$

18.(18)

$3x^2 - 3x + 6\lambda = 0$

$3x^2 - 10x + 27\lambda = 0$

$- \quad + \quad -$

$7x - 21\lambda = 0$ $\qquad \alpha\beta = 2\lambda$

$x = 3\lambda$ (common root) $\qquad \beta = \dfrac{2\lambda}{\alpha} = \dfrac{2\lambda}{3\lambda}$

$9\lambda^2 - 3\lambda + 2\lambda = 0$ $\qquad \alpha\gamma = \dfrac{27\lambda}{3} = 9\lambda$

$9\lambda^2 - \lambda = 0$ $\qquad \gamma = \dfrac{9\lambda}{\alpha} = \dfrac{9\gamma}{3\lambda} = \dfrac{9}{3}$

$\lambda = 0\,;\ \lambda = \dfrac{1}{9}$

Common root $= 3 \times \dfrac{1}{9} = \dfrac{1}{3} = \alpha$

$$\frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3} \times \frac{9}{3}}{\frac{1}{9}} = \frac{\frac{18}{9}}{\frac{1}{9}} = 18$$

19.(66) $\dfrac{2}{x-1} - \dfrac{1}{x-2} = \dfrac{2}{k};\ k \neq 0 \ \Rightarrow \quad \left[2(x-2) - (x-1)\right]k == 2\left[x^2 - 3x + 2\right]$

$\Rightarrow \quad 2x^2 + x(-6-k) + (4+3k) = 0$

Since, we have no real root $\quad \Rightarrow \quad D < 0 \ \Rightarrow 36 + k^2 + 12k - 8(4+3k) < 0$

$\Rightarrow \quad k^2 - 12k + 4 < 0 \Rightarrow 6 - \dfrac{\sqrt{128}}{2} < k < 6 + \dfrac{\sqrt{128}}{2} \ \Rightarrow \quad k \in (0.xx, 11.xx)$

$\Rightarrow \quad$ Sum of integral values of $k = 1 + 2 + 3 + \ldots + 11 = \dfrac{11 \times 12}{2} = 66$

JEE Advanced 2021

1.(4) If $x^2 > 1$; $\qquad 3x^2 - 4x^2 + 4 + x - 1 = 0$

$-x^2 + x + 3 = 0$; $\qquad x^2 - x - 3 = 0$

$x = \dfrac{1 + \pm\sqrt{13}}{2} = \dfrac{1 + \sqrt{13}}{2}$ *or* $\dfrac{1 - \sqrt{13}}{2}$

If $x^2 < 1$

$3x^2 + 4x^2 - 4 + x - 1 = 0$; $\qquad 7x^2 + x - 5 = 0$

$x = \dfrac{1 \pm \sqrt{1 + 140}}{14} = \dfrac{-1 + \sqrt{141}}{14}, \dfrac{-1 - \sqrt{141}}{14}$

Solutions of Archive - JEE Main & Advanced

Quadratic Equations	Class - XI | Mathematics

JEE Main 2022

1.(C) $x^2+(3-a)x+1=2a$

$\Rightarrow x^2+(3-a)x+(1-2a)=0$

α , β *are (roots)*

$\alpha+\beta=(a-3),\ \alpha\beta=\dfrac{1-2a}{1}$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(a-3)^2-2(1-2a) \Rightarrow a^2-6a+9-2+4a$

$f(a)=a^2-2a+7\Rightarrow(a-1)^2+6$

minimum value of $\alpha^2+\beta^2=6$

2.(16) $x^2-x-4=0 \begin{cases}\alpha\\ \beta\end{cases}$

$\alpha^2-\alpha-4=0$ $\boxed{p_n=\alpha^n-\beta^n}$

$\alpha^n-\alpha^{n-1}-4\alpha^{n-2}=0$ ...(1)

11*y* $\beta^n-\beta^{n-1}-4\beta^{n-2}=0$ ...(2)

(1) – (2)

$(\alpha^n-\beta^n)-(\alpha^{n-1}-\beta^{n-1})-4(\alpha^{n-2}-\beta^{n-2})=0$ $\boxed{p_n-p_{n-1}=4p_{n-2}}$

So, $\dfrac{p_{15}p_{16}-p_{14}p_{16}-p_{15}^2+p_{14}p_{15}}{p_{13}p_{14}}$

$\Rightarrow$ $\dfrac{(p_{15}-p_{14})(p_{16}-p_{15})}{p_{13}p_{14}}$ $\Rightarrow$ $\dfrac{(4p_{13})\times(4p_{14})}{p_{13}p_{14}}=16$

3.(B) $(e^{2x}-4)(6e^{2x}-3e^x-2e^x+1)=0$

$(e^{2x}-4)(3e^x-1)(2e^x-1)=0$

$\therefore\ e^{2x}=4, e^x=\dfrac{1}{3},\ e^x=\dfrac{1}{2}$ $\therefore\ x=\dfrac{1}{2}\ln 4,\ x=\ln\dfrac{1}{3},\ x=\ln\dfrac{1}{2}$

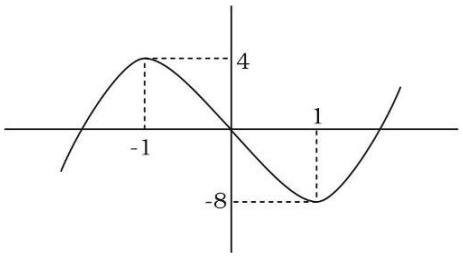
$\therefore$ Sum of real roots $=\dfrac{1}{2}\ln 4+\ln\dfrac{1}{3}+\ln\dfrac{1}{2}=-\ln 3$

4.(D) $x^7 - 7x - 2 = 0$

$f(x) = x^7 - 7x - 2$

$f'(x) = 7x^6 - 7 = 7(x^6 - 1)$ $\quad\therefore\quad f'(x) = 7(x^2 - 1)(x^4 + x^2 + 1)$

$f'(x) = 0 \Rightarrow x = \pm 1$



$\therefore$ $f(x)$ has 3 real roots

5.(B) $\alpha + \beta = \dfrac{-\lambda}{3}$ $\qquad \alpha\beta = \dfrac{-1}{3}$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2} = 15$$

$$\frac{\frac{\lambda^2}{9} + \frac{2}{3}}{\frac{1}{9}} = 15 \Rightarrow \frac{\frac{\lambda^2}{9} + \frac{6}{9}}{\frac{1}{9}} = 15 \Rightarrow \lambda^2 = 9$$

Hence, $6(\alpha^3 + \beta^3)^2 = 6((\alpha+\beta)((\alpha+\beta)^2 - 3\alpha\beta))^2 = 6\left(\left(\frac{\lambda^2}{9}\right)\left(\frac{\lambda^2}{9} + 1\right)\right)^2 = 24$

6.(B) Given $ax^2 - 2bx + 15 = 0$

Root $\overbrace{\alpha \quad \alpha}$

Sum $= 2\alpha = \dfrac{2b}{a} \Rightarrow \alpha = \dfrac{b}{a}$ $\qquad$ ...(i)

Product $= \alpha^2 = \dfrac{15}{a}$ $\qquad$ ...(ii)

$\dfrac{(ii)}{(i)}$ $\dfrac{\alpha^2}{\alpha} = \dfrac{15}{a} \times \dfrac{a}{b} \Rightarrow \boxed{\alpha = \dfrac{15}{b}}$

α also lies on $x^2 - 2bx + 21 = 0$

$\Rightarrow \alpha^2 - 2b\alpha + 21 = 0$

$\Rightarrow \dfrac{225}{b^2} - \dfrac{30b}{b} + 21 = 0$

$\Rightarrow \dfrac{225}{b^2} = 9 \Rightarrow \boxed{b^2 = 25}$

How, $\alpha + \beta,\ \alpha\beta = 21$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 4b^2 - 42$

$= 100 - 42 = 58$

7.(727) $\Delta n_g = 1 - \dfrac{3}{2} = -\dfrac{1}{2}$

8.(98) $x^2 - 4\lambda x + 5 = 0 \langle_{\beta}^{\alpha}$

$x^2 - \left(3\sqrt{2} + 2\sqrt{3}\right)x + \left(7 + 3\lambda\sqrt{3}\right) = 0 \langle_{\gamma}^{\alpha}$

$\alpha + \beta = 4\lambda$

$\alpha + \gamma = 3\sqrt{2} + 2\sqrt{3}$

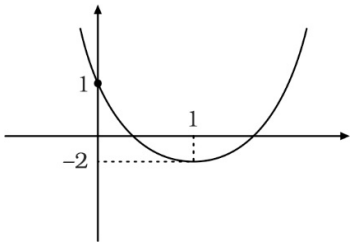
$\beta + \lambda = 3\sqrt{2}$ $\quad \alpha\gamma = 7 + 3\lambda\sqrt{3}$

$\therefore$ $\quad \alpha + 2\lambda + \sqrt{3}$ $\quad \alpha + \beta = 5$

$\beta = 2\lambda - \sqrt{3}$ $\quad 4\lambda^2 = 8 \Rightarrow \lambda = \sqrt{2}$

$\therefore$ $\quad \left(\alpha + 2\beta + \lambda\right)^2 = \left(4\alpha + 3\sqrt{2}\right)^2 = \left(7\sqrt{2}\right)^2 = 98$

9.(B) Let $f\left(x\right) = x^4 - 4x + 1$



$f'\left(x\right) = 4\left(x - 1\right)\left(x^2 + x + 1\right)$

$f\left(x\right) \uparrow x > 1$

$f\left(x\right) \downarrow x < 1$

Two real solution

10.(45) $e^{2x} - 11e^{x} - 45e^{-x} + \frac{81}{2} = 0$

$2e^{3x} - 22e^{2x} - 90 + 81e^{x} = 0$

Let $e^{x} = t$

$2t^3 - 22t^2 + 81t - 90 = 0$; *Routes are* $e^{x_1}, e^{x_2}, e^{x_3}$

$e^{x_1 + x_2 + x_3} = 45$

$x_1 + x_2 + x_3 = \ln 45$ $\quad \therefore P = 45$

11.(A) Let $f\left(x\right) = \left(x + 1\right)\left(ax + b\right)$

$f\left(-2\right) + f\left(3\right) = -1\left(-2a + b\right) + 4\left(3a + b\right) = 0$

$2a - b + 12a + 4b = 0$

$14a + 3b = 0$

$\frac{-b}{a} = \frac{14}{3}$

Sum of roots $= \left(-1 + \frac{-b}{a}\right) = -1 + \frac{14}{3} = \frac{11}{3}$

12.(B) Given equation

$x^7 + 5x^3 + 3x + 1 = 0$

Let $f(x) = x^7 + 5x^3 + 3x + 1$

$f'(x) = 7x^6 + 15x^2 + 3 > 0, \forall x \in R$

Function is an increasing function, and ranges from $(-\infty, \infty)$.

So, there is only one solution.

Solutions of Archive - JEE Main & Advanced

Trigonometry	Class - XI | Mathematics

JEE Main 2021

1.(A) Max. $\left(2^{(6\sin 3x+8\cos 3x)}\right)=2^{10}=\alpha$

Min. $\left(2^{(6\sin 3x+8\cos 3x)}\right)=2^{-10}=\beta$

$\alpha^{1/5}=4, \quad \beta^{1/5}=\frac{1}{4}$

Product of roots $\Rightarrow \ 1=\frac{c}{8} \ \Rightarrow c=8$

Sum of roots $\Rightarrow \ 4+\frac{1}{4}=\frac{-b}{8} \quad \Rightarrow \quad b=-34$

$c-b=42$

2.(A) $\text{cosec}18^\circ=\frac{1}{\sin 18^\circ}=\frac{4}{\sqrt{5}-1}=\sqrt{5}+1$

Let $x=\text{cosec}18^\circ=\sqrt{5}+1$

$x-1=\sqrt{5}$

Squaring both sides

$x^2+1-2x=5$

$x^2-2x-4=0$

3.(B) Given : $t^2-9t+8=0$ To find $\frac{2\sin x}{\sin x+\sqrt{3}\cos x} \ (0<x<\pi/2)$

$=e^{(\cos^2 x+\cos^4 x+\cos^6 x+....)\log_e 2}=\left(e^{\log_e 2}\right)^{(\cos^2 x+\cos^4 x+....)}=2^{(\cos^2 x+\cos^4 x+.....\infty)}$

$=2^{\frac{\cos^2 x}{1-\cos^2 x}}=2^{\frac{\cos^2 x}{\sin^2 x}}=2^{\cot^2 x}$

Now $2^{\cot^2 x}$ satisfies $t^2-9t+8=0$

$t^2-8t-t+8=0$

$t(t-8)-1(t-8)=0$

$t=1,8$

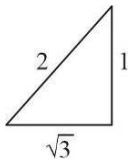
$2^{\cot^2 x}=1 \qquad 2^{\cot^2 x}=8$

$2^{\cot^2 x}=2^0 \qquad 2^{\cot^2 x}=2^3$

$\cot^2 x=0 \qquad \cot^2 x=3$

as $0<x<\pi/2$

$\Rightarrow \ \cot x=\sqrt{3}$



$\sin x = \frac{1}{2},\ \cos x = \frac{\sqrt{3}}{2}$; $\quad \frac{2\sin x}{\sin x + \sqrt{3}\cos x} = \frac{2\left(\frac{1}{2}\right)}{\frac{1}{2} + \sqrt{3}\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\frac{1}{2}+\frac{3}{2}} = \frac{1}{2}$

4.(C) $\tan\theta = \frac{x}{75}$ ...(i)

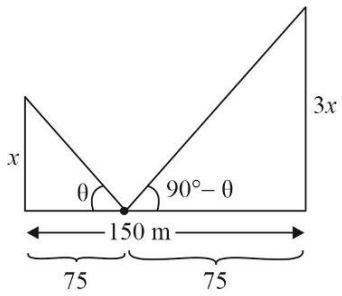
$\tan(90-\theta) = \frac{3x}{75} = \frac{x}{25}$

$\cot\theta = \frac{x}{25}$ ...(ii)

Multiplying equation (i) and (ii)

$\tan\theta \cdot \cot\theta = \frac{x}{75} \times \frac{x}{25}$

$1 = \frac{x^2}{75 \times 25}$; $x^2 = 75 \times 25$; $x = 25\sqrt{3}$



5.(B) $\sin 2\theta\left[1 + \frac{1}{\cos 2\theta}\right] > 0 \Rightarrow \frac{\sin 2\theta}{\cos 2\theta}\left[1+\cos 2\theta\right] > 0 \Rightarrow \tan 2\theta\left(1+\cos 2\theta\right) > 0$

$\Rightarrow \tan 2\theta > 0$ and $\left(1+\cos 2\theta \neq 0\right) \Rightarrow 0 < 2\theta < \frac{\pi}{2}$ or $\pi < 2\theta < \frac{3\pi}{2}$ or $2\pi < 2\theta < \frac{5\pi}{2}$ or $3\pi < 2\theta < \frac{7\pi}{2}$

$\Rightarrow \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

6.(C) $\cos x + \cos y - \cos(x+y) = \frac{3}{2} \quad \Rightarrow \quad 2\cos\frac{x+y}{2}.\cos\frac{x-y}{2} - 2\cos^2\frac{x+y}{2} + 1 - \frac{3}{2} = 0$

$\Rightarrow \quad 2\cos^2\frac{x+y}{2} - 2\cos\frac{x+y}{2}.\cos\frac{x-y}{2} + \frac{1}{2} = 0 \quad \Rightarrow \quad 4\cos^2\frac{x+y}{2} - 4\cos\frac{x+y}{2}.\cos\frac{x-y}{2} + 1 = 0$

$\Rightarrow \quad \cos^2\frac{x+y}{2} - \cos\frac{x+y}{2}.\cos\frac{x-y}{2} + \frac{1}{4}\left(\cos^2\frac{x-y}{2} + \sin^2\frac{x-y}{2}\right) = 0$

$\Rightarrow \quad \left(\cos\frac{x+y}{2} - \frac{1}{2}\cos\frac{x-y}{2}\right)^2 + \frac{1}{4}\left(\sin\left(\frac{x-y}{2}\right)\right)^2 = 0$

$\Rightarrow \quad \sin\frac{x-y}{2} = 0 \quad \& \quad \cos\frac{x+y}{2} = \frac{1}{2}\cos\frac{x-y}{2}$

$\Rightarrow \quad x = y \quad \text{and} \quad \cos x = \frac{1}{2} = \cos y$

$\therefore \quad \sin x = \frac{\sqrt{3}}{2} \quad \therefore \quad \sin x + \cos y = \frac{1+\sqrt{3}}{2}.$

7.(11) $\underline{3\sin x + 4\cos x} = k+1$

Range of $3\sin x + 4\cos x$ is $\left[-5, 5\right]$

$\Rightarrow \quad -5 \le k+1 \le 5 \quad \Rightarrow \quad -6 \le k \le 4$

No. of values of $k = 11$

8.(1) $\sqrt{3}\cos^2 x - \left(\sqrt{3}-1\right)\cos x - 1 = 0$

$\sqrt{3}\cos^2 x - \sqrt{3}\cos x + \cos x - 1 = 0$

$\sqrt{3}\cos x\left(\cos x - 1\right) + \left(\cos x - 1\right) = 0$

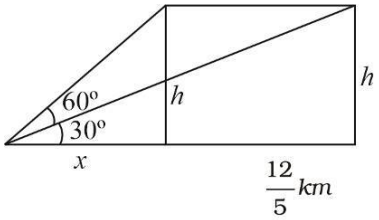
$\left(\cos x - 1\right)\left(\sqrt{3}\cos x + 1\right) = 0$

$\cos x = 1, \cos x = \frac{-1}{\sqrt{3}}$ $\left(\text{No solution in } \left[0, \frac{\pi}{2}\right]\right)$

One solution is possible

9.(C) $\tan 60^\circ = \frac{h}{x} \Rightarrow h = \sqrt{3}x$

$\tan 30^\circ = \frac{h}{x + \frac{12}{5}} \Rightarrow h = \frac{x + \frac{12}{5}}{\sqrt{3}}$

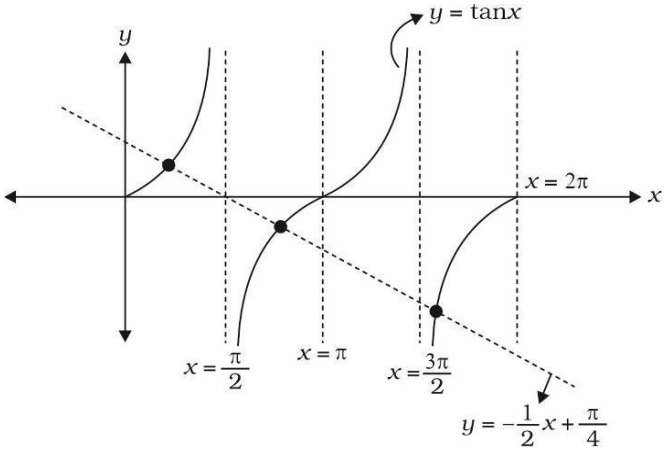


$\Rightarrow \sqrt{3}x = \frac{x + \frac{12}{5}}{\sqrt{3}} \Rightarrow x = \frac{6}{5}km \Rightarrow h = \sqrt{3} \times \frac{6}{5}km = \sqrt{3} \times \frac{6}{5} \times 1000\ m = 1200\sqrt{3}m$

10.(B) $x + 2\tan x = \frac{\pi}{2}\ ;\ x \in [0, 2\pi]$

$\Rightarrow \tan x = \frac{\pi}{4} - \frac{x}{2}$

Let $y = \tan x$ & $y = \frac{\pi}{4} - \frac{x}{2}$ ...(ii)



(i) & (ii) $\Rightarrow$ number of point of intersections are 3.

11.(1) $|\cot x| = \cot x + \frac{1}{\sin x}$

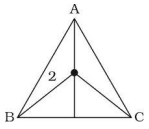
Obviously cot$x$ < 0

$\Rightarrow \quad -\cot x = \cot x + \frac{1}{\sin x}$

$\Rightarrow \quad -\frac{2\cos x}{\sin x} = \frac{1}{\sin x}$

or $\quad \cos x = -\frac{1}{2} \quad \Rightarrow \quad x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

12.(B)



$\tan 60° = \frac{h}{2}$

$h = 2\sqrt{3}$

13.(B) $15\sin^4\alpha + 10\cos^4\alpha = 6\ ;\ \alpha \in R$

$15\sin^4\alpha + 10(1 - \sin^2\alpha)^2 = 6$

$15\sin^4\alpha + 10(1 + \sin^4\alpha - 2\sin^2\alpha) = 6$

$15\sin^4\alpha + 10 + 10\sin^4\alpha - 20\sin^2\alpha - 6 = 0$

$25\sin^4\alpha - 20\sin^2\alpha + 4 = 0$

$(5\sin^2\alpha - 2)^2 = 0$

$\sin^2\alpha = \frac{2}{5} \quad \cos^2\alpha = \frac{3}{5}$

$27\sec^6\alpha + 8\operatorname{cosec}^6\alpha = \frac{27}{(\cos\alpha)^6} + \frac{8}{(\sin\alpha)^6} = \frac{27}{(3/5)^3} + \frac{8}{(2/5)^3} = \frac{27}{27/125} + \frac{8}{8/125} = 125 + 125 = 250$

14.(A) $\because \quad (81)^{\sin^2 x} + (81)^{1-\sin^2 x} = 30$

Now let $(81)^{\sin^2 x} = t \quad \therefore \quad t + \frac{81}{t} = 30 \Rightarrow t^2 - 30t + 81 = 0 \quad \therefore \quad t = 3, 27$

When $t = 3 \Rightarrow 81^{\sin^2 x} = 3^1 \Rightarrow \sin^2 x = \frac{1}{4} \Rightarrow \quad \sin x = \frac{1}{2} (\because x \in [0, \pi]) \quad \Rightarrow \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$

When $t = 27 \Rightarrow 81^{\sin^2 x} = 3^3 \Rightarrow \sin^2 x = \frac{3}{4}$.

$\therefore \quad \sin x = \frac{\sqrt{3}}{2} (\because x \in [0, \pi]) \quad \therefore \quad x = \frac{\pi}{3}, \frac{2\pi}{3} \quad \therefore$ Total number of solutions = 4

15.(D) Since $\cot\left(\frac{\pi}{12}\right) = 2 + \sqrt{3}$

say $\frac{\pi}{24} = \theta \quad \Rightarrow \quad 2\theta = \frac{\pi}{12}$

$\cot(2\theta) = \frac{\cot^2\theta - 1}{2\cot\theta} = 2 + \sqrt{3} \Rightarrow \quad \cot^2\theta - 2(2+\sqrt{3})\cot\theta = 1 \Rightarrow \quad (\cot\theta - (2+\sqrt{3}))^2 = (2+\sqrt{3})^2 + 1$

$(\cot\theta - (2+\sqrt{3}))^2 = 8 + 4\sqrt{3}$

$\cot\theta - (2+\sqrt{3}) = 2\sqrt{2+\sqrt{3}}$

$\cot\theta = 2\sqrt{2+\sqrt{3}} + 2 + \sqrt{3} = \sqrt{8+4\sqrt{3}} + 2 + \sqrt{3} = \sqrt{(\sqrt{2}+\sqrt{6})^2} + 2 + \sqrt{3} = \sqrt{2} + \sqrt{6} + 2 + \sqrt{3}$

16.(A) $\sin\theta + \cos\theta = \frac{1}{2} \quad \Rightarrow 1 + \sin 2\theta = \frac{1}{4}$

$\sin 2\theta = -\frac{3}{4}$

$\cos 4\theta = 1 - 2\sin^2 2\theta = 1 - 2 \times \frac{9}{16} = 1 - \frac{9}{8} = \frac{-1}{8}$

$\cos 4\theta = \frac{-1}{8}$

$\sin 6\theta = 3\sin 2\theta - 4\sin^3 2\theta = 3 \times \frac{-3}{4} - 4 \times \frac{-27}{64} = \frac{27}{16} - \frac{9}{4} = \frac{27-36}{16} = \frac{-9}{16}$

$\sin 6\theta = -\frac{9}{16}$

$16(\sin 2\theta + \cos 4\theta + \sin 6\theta) = 16\left(\frac{-3}{4} - \frac{1}{8} - \frac{9}{16}\right) = -12 - 2 - 9 = -23$

17.(A) $x = \frac{1}{2}, \; y = 0$

$f\left(\frac{1}{2}\right) + f\left(\frac{1}{2}\right) = 2f\left(\frac{1}{2}\right) f(0)$

$-2 = 2(-1)\, f(0) \quad \Rightarrow \quad f(0) = 1$

$x = \frac{1}{2}, \; y = \frac{1}{2}$

$f(1) + f(0) = 2f\left(\frac{1}{2}\right) f\left(\frac{1}{2}\right)$

$f(1) = 1 \Rightarrow \; f(2) = 1 \Rightarrow f(3) = 1$ like wise

$$\sum_{k=1}^{20} \frac{1}{\sin k \sin(k+1)} = \frac{1}{\sin 1} \sum_{k=1}^{20} \frac{\sin(k+1-k)}{\sin k \sin(k+1)}$$

$$\frac{1}{\sin 1} \sum_{k=1}^{20} \cot k - \cot(k+1) = \frac{(\cot 1 - \cot 21)}{\sin 1} = \frac{\sin 20}{\sin 1 \sin 1 \sin 21}$$

18.(B) $2x = \tan\left(\frac{\pi}{9}\right) + \tan\left(\frac{7\pi}{18}\right)$

$2x = \tan(20) + \tan 70$

$2y = \tan\left(\frac{\pi}{9}\right) + \tan\left(\frac{5\pi}{18}\right)$

$2y = \tan 20 + \tan 50$

$x - 2y = \frac{\tan 20 + \tan 70}{2} - \tan 20 - \tan 50 \Rightarrow \quad \frac{\tan 70 - \tan 20 - 2\tan 50}{2}$

$\frac{\cot 20 - \tan 20 - 2\tan 50}{2}$

$\frac{2\cot 40 - 2\tan 50}{2} = 0$

19.(B) $2\cos x[4(\sin^2 45° - \sin^2 x) - 1] = 1$

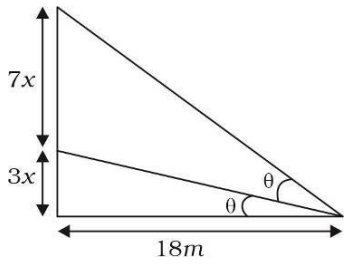
$2\cos x[4\cos^2 x - 3] = 1$

$\cos 3x = 1/2 \quad \therefore x = \pi/9, \ \frac{5\pi}{9}, \ \frac{7\pi}{9}$

$n = 3$

$S = \frac{13\pi}{9}$

20.(B) $\tan\theta = \frac{3x}{18} = \frac{x}{6}$



$\tan 2\theta = \frac{10x}{18} = \frac{5x}{9}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{5x}{9}$

$\frac{2x/6}{1-x^2/36} = \frac{5x}{9} \Rightarrow \frac{3(36)}{36-x^2} = 5$

$5x^2 = 36 \times 2 \Rightarrow \quad x = 6\sqrt{\frac{2}{5}}$

$\text{height} = 10x = 10 \times 6\sqrt{\frac{2}{5}} = 12\sqrt{10}$

21.(56) $\sin^4\theta + \cos^4\theta - \sin\theta.\cos\theta = 0$

$\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta.\cos^2\theta - \sin\theta.\cos\theta = 0$

$1 - \frac{1}{2}\sin^2 2\theta - \frac{1}{2}\sin 2\theta = 0$

$\sin^2 2\theta + \sin 2\theta - 2 = 0, \quad \sin 2\theta = -2, \sin 2\theta = 1$

For $\sin 2\theta = 1$ *in* $\theta \in [0, 4\pi]$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$

$sum = \dfrac{28\pi}{4} = 7\pi$

$\dfrac{8S}{\pi} = \dfrac{8.7\pi}{\pi} = 56$

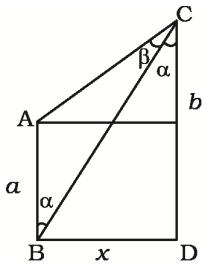
22.(C) $\tan\angle ACB = \dfrac{1}{2}$

$\tan\beta = \dfrac{1}{2}$

$\tan(\alpha+\beta) = \dfrac{x}{b}$

$\tan\alpha = \dfrac{x}{a+b}$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$



$\dfrac{x}{b} = \dfrac{\dfrac{1}{2}+\dfrac{x}{a+b}}{1-\left(\dfrac{1}{2}\right)\left(\dfrac{x}{a+b}\right)} \Rightarrow x^2 - 2ax + ab + b^2 = 0$

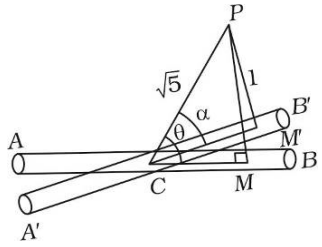
23.(D) $2\left(\dfrac{1}{2}\right)\left(\sin\dfrac{\pi}{8}\sin\dfrac{3\pi}{8}\right)^2 \Rightarrow \dfrac{1}{4}\left[\cos\dfrac{\pi}{4}-\cos\dfrac{\pi}{2}\right]^2 \Rightarrow \dfrac{1}{4}\left[\dfrac{1}{\sqrt{2}}\right]^2 \Rightarrow \dfrac{1}{8}$

24.(D) $\tan\theta = 2$

$\sin\theta = \dfrac{2}{\sqrt{5}} = \dfrac{PM}{\sqrt{5}}$

$PM = 2$

$\sin\alpha = \dfrac{1}{\sqrt{5}}$



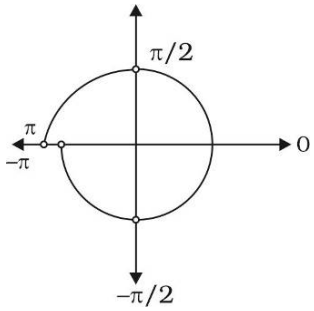
$\Rightarrow \quad \tan\alpha = \dfrac{1}{2}$

Angle of rotation $= \theta - \alpha = \tan^{-1}2 - \tan^{-1}\dfrac{1}{2} = \tan^{-1}\dfrac{3}{4}$

25.(B) $\dfrac{\cos x}{1+\sin x} = \left|\tan 2x\right|;\ x \in \left(\dfrac{-\pi}{2},\dfrac{\pi}{2}\right) - \left(\dfrac{-\pi}{4},\dfrac{\pi}{4}\right)$

$2x \in (-\pi,\pi) - \left\{\dfrac{-\pi}{2},\dfrac{\pi}{2}\right\}$

Case I: $2x \in \left(-\pi,\dfrac{-\pi}{2}\right) \cup [0,\pi/2)$:



$\dfrac{\cos x}{1+\sin x} = \tan 2x = \dfrac{2\sin x\cos x}{1-2\sin^2 x} \quad \left\{\because \tan 2x = \dfrac{\sin 2x}{\cos 2x}\right\}$

$\Rightarrow \quad \cos x\left[1-2\sin^2 x\right] = \cos x\left[2\sin x\left(1+\sin x\right)\right]$

$\Rightarrow \quad \cos x = 0$ (OR) $1 - 2\sin^2 x = 2\sin x + 2\sin^2 x$

$\Rightarrow \quad x \in \phi$ (OR) $4\sin^2 x + 2\sin x - 1 = 0; x \in \left(\frac{-\pi}{2}, \frac{-\pi}{4}\right) \cup [0, \frac{\pi}{4})$

i.e., $\sin x = \frac{\sqrt{5}-1}{4}, -\left(\frac{\sqrt{5}+1}{4}\right) \Rightarrow x = 18°, -54°$

Case II : $2x \in \left(\frac{-\pi}{2}, 0\right) \cup \left(\frac{\pi}{2}, \pi\right)$:

$$\frac{\cos x}{1+\sin x} = -\tan 2x = -\frac{\sin 2x}{\cos 2x} = \frac{-2\sin x \cos x}{1 - 2\sin^2 x}$$

$\Rightarrow \quad \cos x\left[1 - 2\sin^2 x\right] = -\cos x\left[2\sin x(1 + \sin x)\right]$

$\Rightarrow \quad \cos x = 0$ (OR) $1 - 2\sin^2 x = -2\sin x - 2\sin^2 x$

$\Rightarrow \quad x \in \phi$ (OR) $\sin x = -1/2; x \in \left(\frac{-\pi}{4}, 0\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \quad \Rightarrow \quad x = -30°$

Hence, (Case I) $\cup$ (Case II) $\Rightarrow \quad x = 18°, -54°, -30°$

Sum $= -66° = \frac{-11\pi}{30}$

Solutions of Archive - JEE Main & Advanced

Trigonometry	Class - XI | Mathematics

JEE Main 2022

1.(D) $\cos\left(\frac{\pi}{3}+x\right)\cos\left(\frac{\pi}{3}-x\right)=\frac{1}{4}\cos^2 2x$

$4\left(\cos^2\left(\frac{\pi}{3}\right)-\sin^2 x\right)=\cos^2 2x$

$4\left(\frac{1}{4}-\sin^2 x\right)=\cos^2 2x$

$\Rightarrow 1-4\sin^2 x=\cos^2 2x \Rightarrow 1-2(1-\cos 2x)=\cos^2 2x$

Now let $\cos 2x = t$

$\therefore -1+2\cos 2x=\cos^2 2x \qquad \therefore t^2-2t+1=0 \Rightarrow t=1$

$\therefore \cos 2x=1 \qquad \therefore 2x=2n\pi$

$x=n\pi$

$\therefore n=-3,-2,-1,0,1,2,3$ for $x\in[-3\pi,3\pi]$

2.(B) $f'(x)=\begin{cases}4x+3+\frac{\cos 2x}{4}, & \frac{1}{2}<x<1\\ -(4x+3)+\frac{\cos 2x}{4}, & 0\le x<\frac{1}{2}\end{cases}$

$f'(x)<0$ for $x<\frac{1}{2}$

$f'(x)>0$ for $x>\frac{1}{2}$

$\therefore x=\frac{1}{2}$ is point of minima and $x=1$ is point of maxima

$\therefore f\left(\frac{1}{2}\right)+f(1)=3+\frac{1}{2}(1+2\cos(1))\sin(1))$

3.(B) $\sin\theta\tan\theta+\tan\theta=\sin 2\theta$

$\tan\theta[\sin\theta+1]=\frac{2\tan\theta}{1+\tan^2\theta}$

$\tan\theta=0 \Rightarrow \theta=-\pi,0,\pi$

$(\sin\theta+1)=2\cos^2\theta$

$\sin\theta=\frac{1}{2}\Rightarrow \theta=\frac{\pi}{6},\frac{5\pi}{6}$

$n(5)=5$

$T=\cos O+\cos 2\pi+\cos 2\pi+\cos\frac{\pi}{3}+\cos\frac{5\pi}{3}$

$T=4$

$T+n(s)=9$

4.(D) $\sin 12^\circ + \sin 12^\circ - \sin 72^\circ \Rightarrow \sin 12^\circ - 2\sin 30^\circ \cos 42^\circ$

$\Rightarrow \sin 12^\circ - \sin 48^\circ \Rightarrow -2\sin 18^\circ \cos 30^\circ = \frac{\sqrt{3}}{4}\left(1-\sqrt{5}\right)$

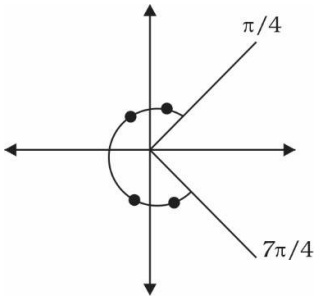
5.(4) $14\,\text{cosec}^2 x - 2\sin^2 x = 21 - 4\cos^2 x$

$\Rightarrow \quad \frac{14}{\sin^2 x} - 2\sin^2 x - 21 + 4\left(1-\sin^2 x\right) = 0$

$\Rightarrow \quad 14 - 2\sin^4 x - 21\sin^2 x + 4\sin^2 x - 4\sin^4 x = 0$

$\Rightarrow \quad 6\sin^4 x + 17\sin^2 x - 14 = 0 \Rightarrow \sin^2 x = \frac{-21}{6}, \frac{2}{3}$

$\Rightarrow \quad \sin x = \pm\sqrt{\frac{2}{3}} = \left\{\begin{matrix} \frac{1}{\sqrt{3/2}} > \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{3/2}} < \frac{-1}{\sqrt{2}} \end{matrix}\right\}$



$\therefore$ For $x \in \left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$, we have four solutions.

6.(B) $16\sin(20^\circ)\sin(40^\circ)\sin(80^\circ)$

We know that $\sin\theta \sin\left(\frac{\pi}{3}-\theta\right)\sin\left(\frac{\pi}{3}+\theta\right) = \frac{1}{4}\sin(3\theta)$

$16\sin(20^\circ)\sin(40^\circ)\sin(80^\circ) = 4\sin(60^\circ) = 2\sqrt{3}$

7.(80) $\sin^2(10^\circ)\sin(20^\circ)\sin(40^\circ)\sin(50^\circ)\sin(70^\circ) = \alpha - \frac{1}{16}\sin(10^\circ)$

Using the fact $\sin\theta(60^\circ-\theta)\sin(60^\circ+\theta) = \frac{1}{4}\sin 3\theta$

We get $\alpha = \frac{1}{64}$

$16 + \alpha^{-1} = 80$

8.(C) $\cos 72^\circ = \frac{\sqrt{5}-1}{4}$

$\Rightarrow \quad 1 - 2\sin^2 36^\circ = \frac{\sqrt{5}-1}{4}$

$\Rightarrow \quad 4 - 8\alpha^2 = \sqrt{5} - 1$

$\Rightarrow \quad 5 - 8\alpha^2 = \sqrt{5}$

$\Rightarrow \quad 25 + 64\alpha^4 - 80\alpha^2 = 5 \quad \Rightarrow \quad 64\alpha^4 - 80\alpha^2 + 20 = 0$

$\Rightarrow \quad 16\alpha^4 - 20\alpha^2 + 5 = 0$

9.(B) $S = \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

$$\cos\alpha + \cos(\alpha+\beta) + \ldots\ldots + \cos\left(\alpha + \overline{n-1}\beta\right) = \frac{\sin n\frac{\beta}{2}}{\sin\frac{\beta}{2}}\cos\left(\alpha + \overline{n-1}\frac{\beta}{2}\right)$$

$$\alpha = \frac{2\pi}{7}; \beta = \frac{2\pi}{7}; n = 3$$

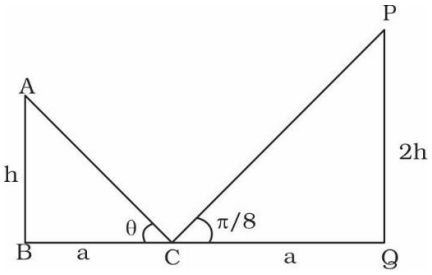
$$S = \frac{\sin 3.\frac{\pi}{7}}{\sin\frac{\pi}{7}} \times \cos\left\{\frac{2\pi}{7} + \frac{2\pi}{7}\right\}$$

$$S = \frac{\sin\frac{3\pi}{7}.\cos\frac{4\pi}{7}}{\sin\frac{\pi}{7}} = \frac{-\sin\frac{3\pi}{7}\cos\frac{3\pi}{7}}{\sin\frac{\pi}{7}} = \frac{-\sin\frac{6\pi}{7}}{2\sin\frac{\pi}{7}} = \frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}} = \frac{-1}{2}$$

10.(A) $\cot\alpha = 1, \sec\beta = \frac{-5}{3}, \cos\beta = \frac{-3}{5}, \tan\beta = \frac{-4}{3}$

$$\tan(\alpha+\beta) = \frac{1-\frac{4}{3}}{1+\frac{4}{3}\times 1} = \frac{-1}{7}$$

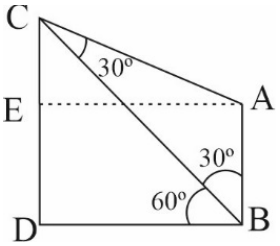
11.(C)



$\tan\theta = \frac{h}{a}$, $\tan\frac{\pi}{8} = \frac{2h}{a}$

$\Rightarrow \tan^2\theta = \frac{1}{4}\tan^2\frac{\pi}{8} = \frac{1}{4}\left(\sqrt{2}-1\right)^2 = \frac{3-2\sqrt{2}}{4}$

12.(D) In the given figure, let *AB* represent the pole and *CD* be the tower.



In $\Delta ABC : \angle ACB = \angle ABC = 30^\circ$

$\Rightarrow AB = AC = 20m$

Also $\angle BAC = 120^\circ \Rightarrow \angle EAC = 30^\circ$

$\Rightarrow CE = 20\sin 30^\circ = 10m$

$\Rightarrow CD = 30m$

13.(4) $\sin x = \cos^2 x = 1 - \sin^2 x$

$\Rightarrow \quad \sin^2 x + \sin x - 1 = 0$

$\Rightarrow \quad \sin x = \frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}$

$\Rightarrow \quad \sin x = \frac{-1+\sqrt{5}}{2}$

$\Rightarrow$ No. of solution in (0, 10) = 4

14.(32) $3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0$

$\Rightarrow \quad 3\cos^2 2\theta + 6\cos 2\theta - 10\left(\frac{1+\cos 2\theta}{2}\right) + 5 = 0$

$\Rightarrow \quad 3\cos^2 2\theta + \cos 2\theta = 0$

$\cos 2\theta = 0 \Rightarrow 2\theta = (2n+1)\frac{\pi}{2} \rightarrow$ 16 solutions of θ in $[-4\pi, 4\pi]$

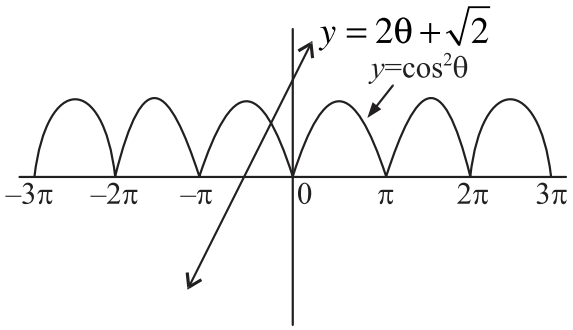
$\cos 2\theta = -\frac{1}{3} \Rightarrow$ 16 solutions of θ in $[-4\pi, 4\pi]$

Thus number of solutions = number of elements in set S = 32

15.(1) $2\theta - \cos^2\theta + \sqrt{2} = 0$

$2\theta + \sqrt{2} = \cos^2\theta$

Number of solutions = Number of point of intersection of graphs $y = \cos^2\theta$ and $y = 2\theta + \sqrt{2} = 1$



16.(B) $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$

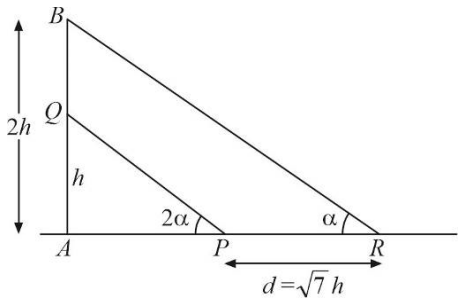
$2\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{3\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\frac{5\pi}{11}\right)$

$= 2\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\frac{8\pi}{11}\right)\cos\left(\frac{16\pi}{11}\right)$

$= 2\frac{\sin\frac{32\pi}{11}}{2^5\sin\frac{\pi}{11}} = \frac{1}{16}$

17.(C) $\tan 2\alpha = \dfrac{h}{AP}$

$AP = h\cot 2\alpha$



$\tan\alpha = \dfrac{AB}{AR}$

$\tan\alpha = \dfrac{2h}{h\cot 2\alpha + \sqrt{7}h} = \dfrac{2}{\cot 2\alpha + \sqrt{7}}$

$\tan\alpha\left(\dfrac{1-\tan^2\alpha}{2\tan\alpha} + \sqrt{7}\right) = 2$

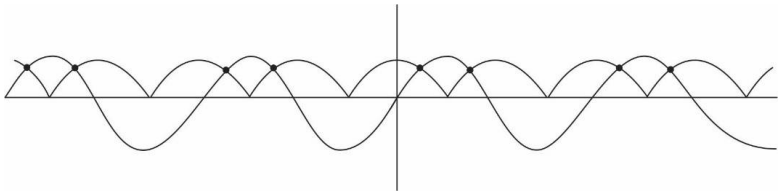
$1 - \tan^2\alpha + 2\sqrt{7}\tan\alpha = 4$

$\tan^2\alpha - 2\sqrt{7}\tan\alpha + 3 = 0$

$\tan\alpha = \dfrac{2\sqrt{7} \pm \sqrt{(2\sqrt{7})^2 - 4\cdot 1\cdot 3}}{2\times 1}$

$= \dfrac{2\sqrt{7}\pm 4}{2} = \sqrt{7}\pm 2$

18.(C)



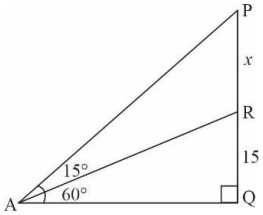
2 soluti0ns in $(0,2\pi)$

So, 8 solutions in $[-4\pi, 4\pi]$

19.(A) $\dfrac{15}{AQ} = \tan 60°$ ...(i)

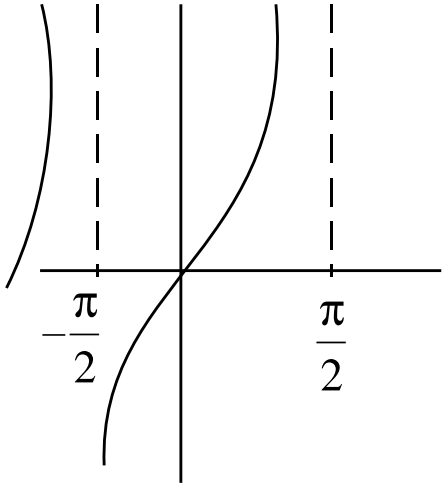
$\dfrac{15+x}{AQ} = \tan 75°$ ...(ii)

$\dfrac{(i)}{(ii)} \Rightarrow x = 10\sqrt{3}$



So, $PQ = 5\left(2\sqrt{3}+3\right)m$

20.(5)

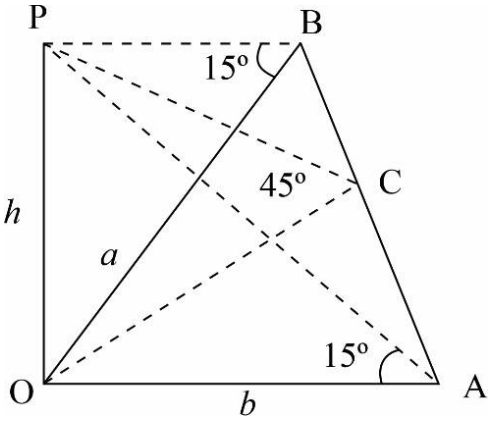


$\tan\theta = x,\ x\left(1+\sqrt{5}\,\frac{2x}{1-x^2}\right) = \sqrt{5} - \frac{2x}{1-x^2}$

$x\,(1-x^2+2\sqrt{5}x) = \sqrt{5} - \sqrt{5}x^2 - 2x$

$x^3 - 3\sqrt{5}x^2 - 3x + \sqrt{5} = 0$ < 2 positive, 1 negative

21.(B)



$\frac{h}{a} = \frac{h}{b} = \tan 15^\circ \Rightarrow a = b$

$OC = \frac{1}{2}\sqrt{2a^2 + 2a^2 - 16^2}$

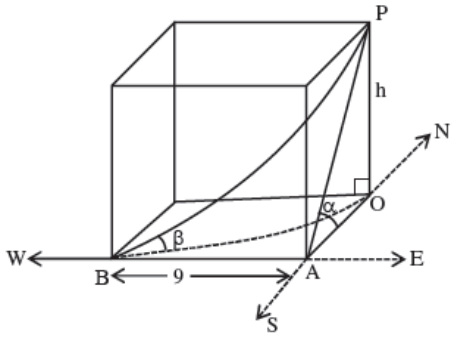
$a\,\tan 15^\circ = \frac{1}{2}\sqrt{4a^2 - 256} = \sqrt{a^2 - 64}$

$\left(2-\sqrt{3}\right)^2 a^2 = a^2 - 64$

$\left(6 - 4\sqrt{3}\right)a^2 = -64$

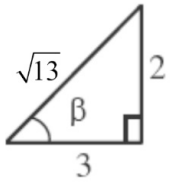
$a^2 = \frac{64}{4\sqrt{3} - 6}$

22.(A)

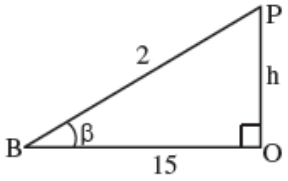


Given $OB = 15$

$\cos\beta = \frac{3}{\sqrt{13}}$



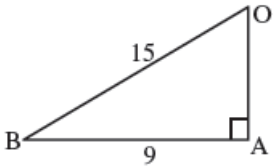
$\tan\beta = \frac{2}{3}$



$\tan\beta = \frac{h}{15}$

$\frac{2}{3} = \frac{h}{15}$

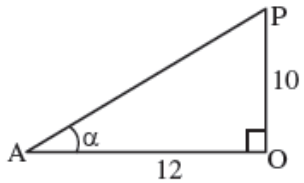
$10 = h$



$OA^2 + AB^2 = 225$

$OA^2 + 81 = 225$

$OA = 12$



$\tan\alpha = \frac{10}{12}$

$\cot\alpha = \frac{12}{10} = \frac{6}{5}$

23.(16) $7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2$

$4\cos^2\theta + 3\cos 2\theta - 2\cos^2 2\theta = 2$

$2(1+\cos 2\theta) + 3\cos 2\theta - 2\cos^2 2\theta = 2$

$2\cos^2 2\theta - 5\cos 2\theta = 0$

$\cos 2\theta(2\cos 2\theta - 5) = 0$

$\cos 2\theta = 0$

$2\theta = (2n+1)\frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{4}$

$S = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

For all four values of θ

$x^2 - 2\left(\tan^2\theta + \cos^2\theta\right)x + 6\sin^2\theta = 0$

$\Rightarrow \quad x^2 - 4x + 3 = 0$

Sum of roots of all four equations = $4 \times 4 = 16$

24.(3) Equation 1 $2\sin^2\theta - \cos 2\theta = 0$

$2\sin^2\theta - (1 - 2\sin^2\theta) = 0 \Rightarrow 4\sin^2\theta = 1$

$\sin\theta = \pm\frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ (1) in $[0, 2\pi]$

Equation 2 $2\cos^2\theta + 3\sin\theta = 0$

$2(1 - \sin^2\theta) + 3\sin\theta = 0$

$2\sin^2\theta - 3\sin\theta - 2 = 0$

$2\sin^2\theta = 4\sin\theta + \sin\theta - 2 = 0$

$2\sin\theta(\sin\theta - 2) + 1(\sin\theta - 2) = 0$

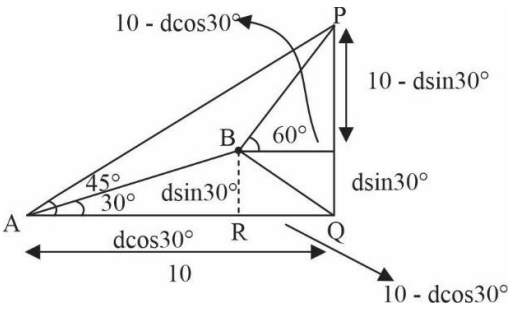
$\sin\theta = 2$ (reject), $-\frac{1}{2}$

$\therefore \sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ (2) in $[0, 2\pi]$

sum of common solutions $\frac{7\pi}{6}, \frac{11\pi}{6} = 3\pi$

25.(A)



$\tan 60° = \dfrac{10 - \frac{d}{2}}{10 - \frac{d\sqrt{3}}{2}} = \sqrt{3}$

$20-d=\sqrt{3}\left(20-d\sqrt{3}\right)$

$2d=20\left(\sqrt{3}-1\right)$

$d=10\left(\sqrt{3}-1\right)$

$Area=\frac{1}{2}\times\left(10-d\cos 30^\circ\right)\left(10+d\sin 30^\circ\right)$

$=\frac{1}{2}\times\left(\sqrt{3}-1\right)5\times 5\left(\sqrt{3}+1\right)=\frac{25}{2}\times 2=25$

26.(C)

$\sum\sec\left(\theta+\alpha-\frac{\pi}{6}\right)\sec\left(\theta+\alpha\right)$

$=\frac{\sin\left[\left(\theta+\alpha\right)-\left(\theta+\alpha-\frac{\pi}{6}\right)\right]}{\cos\left(\theta+\alpha-\frac{\pi}{6}\right)\times\cos\left(\theta+\alpha\right)}$

$S=\sum_{m=1}^{9}\tan\left(\theta+\alpha\right)-\tan\left(\theta+\alpha-\frac{\pi}{6}\right)$

$S=\sum_{m=1}^{9}\tan\left(\theta+\frac{m\pi}{6}\right)-\tan\left(\theta+\frac{m\pi}{6}-\frac{\pi}{6}\right)$

$t_1=\tan\left(\theta+\frac{\pi}{6}\right)-\tan\left(\theta+\frac{\pi}{6}-\frac{\pi}{6}\right)$

$t_2=\tan\left(\theta+\frac{2\pi}{6}\right)-\tan\left(\theta+\frac{2\pi}{6}-\frac{\pi}{6}\right)$

.....

.....

$t_9=\tan\left(\theta+\frac{9\pi}{6}\right)-\tan\left(\theta+\frac{9\pi}{6}-\frac{\pi}{6}\right)$

$S=\tan\left(\theta+\frac{3\pi}{2}\right)-\tan\theta=-\frac{4}{\sqrt{3}}$

$\cot\theta+\tan\theta=\frac{4}{\sqrt{3}}$

$\theta=\frac{\pi}{3}$

27.(C) $8^{2\sin^2\theta}+8^{2-2\sin^2\theta}=16$

$y+\frac{64}{y}=16$

$\Rightarrow y=8 \qquad \Rightarrow \sin^2\theta=1/2$

$n\left(S\right)+\sum_{\theta\in S}\frac{1}{\cos\left(\pi/4+2\theta\right)\sin\left(\pi/4+2\theta\right)}=4+\left(-2\right)\times 4=-4$

Solutions of Archive - JEE Main & Advanced

Sequence and Series	Class - XI | Mathematics

JEE Main 2021

1.(B) $\frac{a+2+a}{3}=\frac{10}{3} \Rightarrow \quad a=4$

Let $b=a+d=4+d$

$c=a+2d=4+2d$

$\frac{c+b+b}{3}=\frac{7}{3} \Rightarrow 12+4d=7 \Rightarrow d=-\frac{5}{4} \quad \therefore \quad b=4+d=4-\frac{5}{4}=\frac{11}{4}$

$\alpha^2+\beta^2-a\beta=(\alpha+\beta)^2-3\alpha\beta=\frac{b^2}{a^2}-\frac{3}{a}=\frac{121}{16\times16}-\frac{3\times64}{4\times64}=\frac{121-192}{256}=-\frac{71}{256}$

2.(3) $a+ar+ar^2+ar^3=\frac{65}{12}$

$\Rightarrow \quad a(1+r+r^2+r^3)=\frac{65}{12}$ .....(1)

$\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+\frac{1}{ar^3}=\frac{65}{18}$

$\Rightarrow \quad \frac{1}{a}\left[\frac{1+r+r^2+r^3}{r^3}\right]=\frac{65}{18}$ .....(2)

Using(1)

$\Rightarrow \quad \frac{1}{a}\times\frac{1}{r^3}\times\frac{65}{12\times a}=\frac{65}{18}$

Also given $a^3r^3=1 \Rightarrow ar=1 \quad \Rightarrow \quad r=\frac{3}{2}$

3rd term $=ar^2=ar\times r$

$=1\times r=\frac{3}{2}$

$2\alpha=3$

3.(D) $x=\frac{1}{1-\cos^2\theta}$; $y=\frac{1}{1-\sin^2\phi} \Rightarrow \cos^2\theta=1-\frac{1}{x}$; $\sin^2\phi=1-\frac{1}{y}$

Now, $z=\frac{1}{1-\cos^2\theta\sin^2\phi} \Rightarrow \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)}=\frac{xy}{xy-(x-1)(y-1)}=\frac{xy}{xy-(xy-x-y+1)}$

$\Rightarrow \quad z=\frac{xy}{x+y-1} \Rightarrow zx+zy-z=xy \Rightarrow z(x+y)=z+xy$

4.(9) $A_1=12$; $A_2=\frac{12}{\sqrt{2}}$; $A_3=\frac{12}{2}$ .... and $A_n=\frac{12}{2^{\frac{n-1}{2}}}$

So, Area of $A_n=\frac{12\times12}{2^{n-1}}<1 \Rightarrow 144<2^{n-1} \Rightarrow n=9$

5.(A) Apply $AM \geq GM = \dfrac{a^{a^x} + \dfrac{a}{a^{a^x}}}{2} \geq \sqrt{a^{a^x} \times \dfrac{a}{a^{a^x}}} \quad \Rightarrow \quad a^{a^x} + a^{1-a^x} \geq 2\sqrt{a}$

6.(C) Let the g.p. be $(a > 0, r > 1)$ in G.P.

$a, ar, ar^2, ar^3, ar^4, ar^5, ar^6, ...$

Given $T_2 + T_6 = ar + ar^5 = \dfrac{25}{2}$ .....(i)

$T_3 \cdot T_5 = ar^2 \cdot ar^4 = 25, a^2r^6 = 25$

$ar^3 = 5$ .....(ii)

In (i)

$ar^3\left(r^{-2} + r^2\right) = \dfrac{25}{2} \quad \Rightarrow \quad 5\left(r^{-2} + r^2\right) = \dfrac{25}{2}$

$r^2 + \dfrac{1}{r^2} = \dfrac{5}{2} \quad \Rightarrow \quad r^2 = 2, r = \sqrt{2}$

$a = \dfrac{5}{2\sqrt{2}}$

$T_4 + T_6 + T_8 = ar^3 + ar^5 + ar^7 = ar^3\left(1 + r^2 + r^4\right) = 5 \cdot (1 + 2 + 4) = 35$

7.(D) Given series

$S = 1 + \dfrac{2}{3} + \dfrac{7}{9} + \dfrac{12}{27} + \dfrac{17}{81} + ...$

$\dfrac{1}{3}S = \dfrac{1}{3} + \dfrac{2}{9} + \dfrac{7}{27} + \dfrac{12}{81} + ...$

$\dfrac{2}{3}S = 1 + \dfrac{1}{3} + \dfrac{5}{9} + \dfrac{5}{27} + \dfrac{5}{81} + ... = \dfrac{4}{3} + \dfrac{5/9}{1 - \dfrac{1}{3}} = \dfrac{4}{3} + \dfrac{5}{6} \quad \Rightarrow \quad S = 2 + \dfrac{5}{4} = \dfrac{13}{4}$

8.(C) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!}$

$(2n+1) = x \Rightarrow \quad n = \dfrac{x-1}{2}$

$$\sum_{x=3,5,7\ldots}^{\infty} \frac{\left(\frac{x-1}{2}\right)^2 + 6\left(\frac{x-1}{2}\right) + 10}{x!} = \sum_{x=3,\ldots.}^{\infty} \frac{x^2 + 10x + 29}{4x!} = \sum \frac{x(x-1) + 11x + 29}{4x!}$$

$$= \frac{1}{4}\left\{\left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} \ldots\ldots\ldots\ldots\right) + 11\left(\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots\ldots\ldots\ldots\right) + 29\left(\frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + \ldots\ldots\right)\right\}$$

$$= \frac{1}{4}\left\{\frac{e - \frac{1}{e}}{2} + 11\left(\frac{e + \frac{1}{e} - 2}{2}\right) + 29\left(\frac{e - \frac{1}{e} - 2}{2}\right)\right\} = \frac{1}{8}\left(41e - \frac{19}{e} - 80\right) = \frac{41}{8}e - \frac{19}{8}e^{-1} - 10$$

9.(10) $-16, 8, -4, 2, - - - - - - G.P.$

$A = -16 \qquad r = \dfrac{8}{-16} = -\dfrac{1}{2}$

$T_p = -16\left(\dfrac{-1}{2}\right)^{p-1} = 32 \times \left(\dfrac{-1}{2}\right)^p$

$T_q = 32\left(\dfrac{-1}{2}\right)^q$

$AM = 16\left(\left(-\frac{1}{2}\right)^p + \left(-\frac{1}{2}\right)^q\right)$

$GM = 32\sqrt{\left(\frac{-1}{2}\right)^{p+q}}$

$4x^2 - 9x + 5 = 0$

roots are $4x^2 - 4x - 5x + 5 = 0$

$4x(x-1) - 5(x-1) = 0$

$x = \frac{5}{4}, 1$

$GM = 1$

$32\sqrt{\left(\frac{-1}{2}\right)^{p+q}} = 1$

$\left(-\frac{1}{2}\right)^{p+q} = \left(-\frac{1}{2}\right)^{10}$

$p + q = 10$

10.(2) $2\log_{10}\left(4^x - 2\right) = 1 + \log_{10}\left(4^x + \frac{18}{5}\right)$

$(4^x - 2)^2 = 10\left(4^x + \frac{18}{5}\right)$

$4^x = t$

$t^2 + 4 - 4t = 10t + 36$

$t^2 - 14t - 32 = 0$

$t^2 - 16t + 2t - 32 = 0$

$t(t-16) + 2(t-16) = 0$

$(t+2)(t-16) = 0$

$\Rightarrow t = 16 \Rightarrow 4^x = 16 = 4^2 \Rightarrow x = 2$

$\begin{vmatrix} 3 & 1 & 4 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{vmatrix} = 3(0-2) - 1(0-4) + 4(1-0) = -6 + 4 + 4 = 2$

11.(C) (100) (100) + (99) (101) + (99) (102) + ...+ (1) (199)

$\sum_{r=0}^{99}(100-r)(100+r) = \sum_{r=0}^{99}(100-r)(100+r)$

$= \sum_{r=0}^{99}(100^2 - r^2) = \sum_{r=0}^{100}(100^2 - r^2)$

$= (100^2).100 - \frac{99(100)(199)}{6} = (100)^2 - (101) - \frac{(100)(101)(201)}{6}$

$= 100^3 - (33)(50)(199)$

$\alpha = 3\ ;\ \beta = (33)(50)$

Slope Slope $= \frac{\beta}{\alpha} = 550$

12.(B) $\sum_{r=1}^{100}\frac{1}{(2r+1)^2 - 1} = \sum_{r=1}^{100}\frac{1}{(2r)(2r+2)} = \frac{1}{2}\sum_{r=1}^{100}\left(\frac{1}{2r} - \frac{1}{2r+2}\right)$

$= \frac{1}{2}\left(\frac{1}{2} - \frac{1}{202}\right) = \frac{1}{2}\cdot\frac{200}{2.202} = \frac{25}{101}$

13.(14) By the condition $\frac{1}{16}$, a and b are in G.P.

$a^2 = \frac{b}{16} \Rightarrow 16a^2 = b$

And it is also given that $\frac{1}{a}, \frac{1}{b}$, 6 are in A.P.

$\frac{2}{b} = \frac{1}{a} + 6$; $\frac{2}{16a^2} = \frac{1}{a} + 6$; $a = \frac{1}{12}$ and $a = -\frac{1}{4}$

Given that $a > 0, so\ a = \frac{1}{12}$; $b = \frac{1}{9}$

Now value of $72(a+b) = 14$

14.(16) $S_n(x) = \log_a x\{2 + 3 + 6 + 11 + 18 + 27 + ... + n\ term\}$

Let $S = 2 + 3 + 6 + 11 + ... + n\ term$

Then $S = 2n + \frac{n(n-1)(2n-1)}{6}$; So $S_n(x) = \left\{2n + \frac{n(n-1)(2n-1)}{6}\right\}\log_a x$

It is given that $S_{24}(x) = 1093$

$\left(48 + \frac{24 \times 23 \times 47}{6}\right)\log_a x = 1093$; $\log_a x = \frac{1}{4}$

$x = a^{1/4}$ ... (i)

Now $S_{12}(2x) = 265$

$(\log_a 2x)\left(24 + \frac{11.12.23}{6}\right) = 265$; $\log_a 2x = \frac{1}{2} = 1,\ 2x = a^{1/2}$

$2a^{1/4} = a^{1/2}$; $16a = a^2$; $a = 0\ and\ a = 16$

But $a \neq 0.\ so\ a = 16$

15.(B) $S_{4n} - S_{2n} = 1000 \Rightarrow \frac{4n}{2}(2a + (4n-1)d - \frac{2n}{2}(2a + (2n-1)d) = 1000$

$\Rightarrow 4an + 2nd(4n-1) - 2an - nd(2n-1) = 1000$

$2an + 6n^2d - nd = 1000$

$\frac{6n}{2}(2a + (6n-1)d) = 3000$

$S_{6n} = 3000$

16.(3) G.P. : 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192

A.P. : 11, 16, 21, 26, 31, 36 .....

Term 256, will be 50th term of A.P.

Common terms are 16, 256, 4096 only $\therefore$ No. of common terms = 3

17.(A) $2\log_9 x + 3\log_9 x + 4\log_9 x + ... + 22\log_9 x = 504$

$\log_9 x\left[2 + 3 + 4 + ... + 22\right] = 504 \Rightarrow \log_9 x \frac{21}{2} \times \left[2 + 22\right] = 504$

$\Rightarrow \log_9 x \frac{21}{2} \times \left[2 + 22\right] = 504 \Rightarrow \log_9 x\ 21 \times 12 = 504$

$\log_9 x = \frac{504}{21 \times 12} = 2$

$x = 9^2 \Rightarrow x = 81$

18.(9) $\dfrac{1}{\alpha(\alpha+1)\ldots(\alpha+20)} = \dfrac{A_0}{\alpha} + \dfrac{A_1}{\alpha+2} + \ldots + \dfrac{A_{20}}{\alpha+20}$

$1 = A_0(\alpha+1)\ldots(\alpha+20) + A_1\alpha(d+2)\ldots(\alpha+20) + \ldots + A_{20}\alpha(\alpha+1)\ldots(\alpha+19)$

Put $\alpha = -14$ $\quad A_{14} = \dfrac{1}{(-14)(-13)\ldots\ldots -1.2\ldots 6}$

Put $\alpha = -15$ $\quad A_{15} = \dfrac{1}{(-15)(-14)\ldots(-1)1\,2\ldots 5}$

$\alpha = 13$ $\quad A_{13} = \dfrac{1}{(-13)(-12)\ldots(-1)\;1\ldots 7}$

$\left(\dfrac{A_{14}+A_{15}}{A_{13}}\right)^2 = \left(\dfrac{\dfrac{1}{-14}\times\dfrac{1}{6} + \dfrac{1}{(-15)(-14)}}{\dfrac{1}{6}\cdot\dfrac{1}{7}}\right)^2 = \left(\dfrac{1}{5} - \dfrac{1}{2}\right)^2 = \dfrac{9}{100}$

19.(7) $S = \left[\dfrac{a_1}{2^3} + \dfrac{a_2}{2^6} + \dfrac{a_3}{2^9} + \ldots\right]$ $\quad \dfrac{S}{2^6} = \left[\dfrac{a_1}{2^9} + \dfrac{a_2}{2^{12}} + \ldots\right]$

$S\left(1 - \dfrac{1}{64}\right) = \left[\dfrac{1}{2^3} + \dfrac{1}{2^6} + \dfrac{a_3 - a_1}{2^9} + \dfrac{a_4 - a_2}{2^{12}}\ldots\right]$

$S\dfrac{63}{64} = \left[\dfrac{1}{8} + \dfrac{1}{64} + \dfrac{2a_2}{2^9} + \dfrac{2a_3}{2^{12}} + \ldots\right]$

$S\dfrac{63}{64} = \left[\dfrac{8+1}{64} + \dfrac{1}{2^2}\left(S - \dfrac{1}{2^3}\right)\right] \Rightarrow S\dfrac{63}{64} = \left[\dfrac{9}{64} + \dfrac{5}{4} - \dfrac{1}{32}\right]$ $\quad 635–165 = 7$

$\Rightarrow \quad S\dfrac{63}{64} = \left(\dfrac{9+165-2}{64}\right)$ $\quad 475 = 7$

20.(1251)

$2040 = 2^3 \times 3^1 \times 5^1 \times 17$

$\therefore$ Required numbers must not be the multiple of 2,3,5 & 17

Let $\quad n(A) =$ sum of number divisible by 2

$n(B) =$ sum of number divisible by 3

$n(C) =$ sum of number divisible by 5

$n(D)$ = sum of number divisible by 17

Required Ans $= (1+2+3+\ldots.+100) - \left[n(A \cup B \cup C \cup D)\right]$

$= \dfrac{100\times 101}{2} - \left[\sum n(A) - \sum n(A \cap B) + \sum n(A \cap B \cap C) - n(A \cap B \cap C \cap D)\right]$

$$= 5050 - \begin{bmatrix} n(A)+n(B)+n(C)+n(D) \\ -n(A\cap B)-n(A\cap C)-n(A\cap D)-n(B\cap C)-n(B\cap D)-n(C\cap D) \\ +n(A\cap B\cap C)+n(A\cap B\cap D)+n(A\cap C\cap D)+n(B\cap C\cap D) \\ -n(A\cap B\cap C\cap D) \end{bmatrix}$$

$$= 5050 - \begin{bmatrix} (2+4+6+\ldots.+100)+(3+6+9+\ldots.+99)+(5+10+15+\ldots.+100)+(17+34+\ldots.+85) \\ -(6+12+18+\ldots\ldots.+96)-(10+20+\ldots.+100)-(34+68)-(15+30+\ldots+90)-(51)-(85) \\ +(30+60+90)+(0)+(0)+(0) \\ -(0) \end{bmatrix}$$

$= 1251$

21.(C) Let a = first term of A.P

d = common difference of AP

$S_{3n} = 3S_{2n}$

$$\frac{3n}{2}\left[2a+(3n-1)d\right]=\frac{3\times 2n}{2}\left[2a+(2n-1)d\right]$$

$$2a+(3n-1)d=4a+(4n-2)d$$

$$0=2a+(n-1)d$$

Now $$\frac{S_{4n}}{S_{2n}}=\frac{\frac{4n}{2}\left(2a+(4n-1)d\right)}{\frac{2n}{2}\left(2a+(2n-1)d\right)}=2\left(\frac{-(n-1)d+(4n-1)d}{-(n-1)d+(2n-1)d}\right)=2\left(\frac{3nd}{nd}\right)=6$$

22.(3) Let $A=1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+.....\infty$

$$B=\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+.....\infty$$

So $I=A^{\log_{0.25}B}$

$$B=\frac{1/3}{1-\frac{1}{3}}=\frac{1}{2}$$

$$A=1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+.....\infty$$

$$\frac{A}{3}=\frac{1}{3}+\frac{2}{3^2}+\frac{6}{3^3}+.....\infty$$

$$\frac{2A}{3}=4\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+.....\infty\right)$$

$$\frac{2A}{3}=4\left(\frac{1/3}{1-\frac{1}{3}}\right)\Rightarrow A=3$$

$$I=(3)^{\log_{0.25}(0.5)}=\sqrt{3}$$

$$I^2=3$$

23.(D) $$\sum_{n=8}^{100}\left[\frac{(-1)^n\, n}{2}\right]=\sum_{m=4}^{50}\left[(-1)^{2m}\,\frac{(2m)}{2}\right]+\sum_{m=4}^{49}\left[(-1)^{2m+1}\,\frac{(2m+1)}{2}\right]=\sum_{m=4}^{50}m+\sum_{m=4}^{49}-(m+1)$$

$$=\sum_{m=4}^{49}m-\sum_{m=4}^{49}m+50-46=4$$

24.(3) $$2\log_3(2^X-5)=\log_3 2+\log_3\left(2^X-\frac{7}{2}\right)$$

Let $2^X=t$

$$\log_3(t-5)^2=\log_3 2\left(t-\frac{7}{2}\right)$$

$(t-5)^2 = 2t-7$

$t^2 - 12t + 32 = 0$

$(t-4)(t-8) = 0 \Rightarrow 2^x = 4$ *or* $2^x = 8$

$x = 2$ (Rejected)

Or $x = 3$

25.(B) $S_n = \sum_{r=1}^{n-1} r\cdot(n-r) = \frac{n(n-1)(n+1)}{6}$

$$\sum_{n=4}^{\infty} \frac{1}{3(n-3)!} = \frac{1}{3}\left[\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+....\right] = \frac{e-1}{3}$$

26.(B) $a_1, a_2, a_3, \_\_\_\_\_ \_\_\_\_ a_{21} \rightarrow$ A.P.

$$\sum_{n=1}^{20} \frac{1}{d}\left[\frac{a_{n+1}-a_n}{a_n\, a_{n+1}}\right] = \frac{4}{9} \quad \Rightarrow \quad a(a+20d) = 45 \rightarrow (i)$$

and $S_{21} = 189 \quad \Rightarrow \quad a+10d = 9 \rightarrow (ii)$

$\therefore \quad a = 15$ and $d = -3/5$

or $\quad a = 3$ and $d = 3/5 \quad \therefore \quad a_6 \cdot a_{16} = 72$

27.(398) $7\times 8, 10\times 10, 13\times 12, 16\times 14.....$

$T_n = (3n+4)(2n+6) = 2(3n+4)(n+3)$

$= 2(3n^2+13n+12) = 6n^2+26n+24$

$$S_{10} = \sum_{n=1}^{10} T_n = 6\sum_{n=1}^{10} n^2 + 26\sum_{n=1}^{10} n + 24\sum_{n=1}^{10} 1$$

$$= \frac{6(10\times 11\times 21)}{6} + 26\times\frac{10\times 11}{2} + 24\times 10 = 3980$$

$$\text{Mean} = \frac{S_{10}}{10} = \frac{3980}{10} = 398$$

28.(D) $T_n = \frac{1+2n}{n^2(n+1)^2} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$

$S_n = \sum_{n=1}^{10} T_n$

$T_1 = \frac{1}{1^2} - \frac{1}{2^2}$

$T_2 = \frac{1}{2^2} - \frac{1}{3^2}$

⋮

$T_n = \frac{1}{n^2} - \frac{1}{(n+1)^2}$

$S_n = \frac{1}{1^2} - \frac{1}{(n+1)^2} \Rightarrow S_{10} = \frac{1}{1} - \frac{1}{11^2} = \frac{120}{121}$

$n = 10$

29.(B) Let numbers be $\frac{a}{r}, a, ar \rightarrow G.P.$

$\frac{a}{r}, 2a, ar \rightarrow A.P.$

$2(2a) = ar + \frac{a}{r}$

$4a = ar + \frac{a}{r}$

$r + \frac{1}{r} = 4$

$r^2 - 4r + 1 = 0$

$r = 4 \pm \frac{\sqrt{16-4}}{2} \quad = 4 \pm \frac{\sqrt{12}}{2} \quad = 2 \pm \sqrt{3}$

4th term in G.P. $= ar^2 = 3r^2$

$a = 3$

$r = 2+\sqrt{3},\ a = 3,\ d = 2a - \frac{a}{r} \quad = 6 - \frac{3}{2+\sqrt{3}}$

$d = 6 - 3(2-\sqrt{3})$

$d = 3\sqrt{3}$

$r^2 - d = \left(2+\sqrt{3}\right)^2 - 3\sqrt{3} \quad = 4 + 3 + 4\sqrt{3} - 3\sqrt{3} \quad = 7 + \sqrt{3}$

30.(C) $\because \quad \frac{2+4+6+\ldots+2y}{3+6+9+\ldots+3y} = \frac{4}{\log_{10} x} \Rightarrow \frac{2\left(1+2+3+\ldots.+y\right)}{3\left(1+2+3+\ldots.+y\right)} = \frac{4}{\log_{10} x} \quad \Rightarrow \quad \log_{10} x = 6 \Rightarrow x = 10^6$

Also, $y = \left(\log_{10} x\right) + \left(\log_{10} x^{\frac{1}{3}}\right) + \left(\log_{10} x^{\frac{1}{9}}\right) + \ldots\ldots\infty$

$= \left(1 + \frac{1}{3} + \frac{1}{9} + \ldots\ldots.\infty\right)\left(\log_{10} x\right) \quad \Rightarrow \quad y = \left(\frac{1}{1-\frac{1}{3}}\right)\log_{10} x = 9 \quad \therefore \quad \left(x,y\right) \equiv \left(10^6, 9\right)$

31.(7744)

A number is divisible by 11 if difference of sum of digits at even places and sum of digits at odd places is divisible by 11.

Possible cases are:

209 + 220 + 242 + 253 + 264 + 275 + 286 + 297 + 308 + 330 + 352 + 363 + 374 + 385 + 396 + 407 + 429 + 440 + 462 + 473 + 484 + 495 = 7744

32.(2021)

$d = -3$ $\quad c_2 = a_2 + b_2 \quad\quad c_3 = 13$

$r = 2$ $\quad a_1 - 3 + b_1 \cdot 2^1 = 12 \quad\quad a_3 + b_3 = 13$

$a_1 + 2b_1 = 15 \quad \ldots\ldots(i) \quad\quad a_1 - 6 + b_1 \cdot 2^2 = 13$

$a_1 + 4b_1 = 19 \quad \ldots\ldots(ii) \quad\quad a_1 + 4b_1 = 19 \quad \ldots\ldots(ii)$

$-2b_1 = -4 \quad \Rightarrow \quad \boxed{b_1 = 2} \quad \boxed{a_1 = 11}$

$\sum_{K=1}^{10} C_k = (a_1 + a_2 + \ldots.. + a_{10}) + (b_1 + b_2 + \ldots.. + b_{10})$

$= \frac{10}{2}[2\times 11 + 9(-3)] + 2\left[\frac{2^{10}-1}{2-1}\right] = 5[22-27] + 2(1023) = -25 + 2046 = 2021$

33.(136) ${}^1P_1 + 2.{}^2P_2 + 3.{}^3P_{3+...+15.}{}^{15}P_{15} = {}^qp_r - s;\ 0 \le s \le 1$

$\Rightarrow \quad 1!+2.(2!)+3.(3!)+...+15(15!) = {}^qp_r - s \qquad \left\{\because\ {}^nP_n = \dfrac{n!}{(n-n)!} = n!\right\}$

$\Rightarrow \quad \sum_{r=1}^{15} r.(r!) = {}^qp_r - s \ \Rightarrow\ \sum_{r=1}^{15}\left[(r+1)r!-(r!)\right] = {}^qp_r - s \ \Rightarrow\ \sum_{r=1}^{15}((r+1)!-r!) = {}^qp_r - s$

$\Rightarrow \quad \begin{Bmatrix} 2!-1! \\ +3!-2! \\ +4!-3! \\ \vdots \\ +16!-15! \end{Bmatrix} = {}^qp_r - s \Rightarrow 16!-1! = {}^qp_r - s \ \Rightarrow\ {}^{16}P_{16} - 1 = {}^qp_r - s$

$\Rightarrow \quad q = r = 16 \text{ and } s = 1 \ \Rightarrow\ {}^{q+s}C_{r-s} = {}^{17}C_{15} = 136$

34.(A) $a + ar + ar^2 + ...\infty = 15 \Rightarrow \dfrac{a}{1-r} = 15 \qquad ...(i)$

$a^2 + (ar)^2 + (ar^2)^2 + ...\infty = 150 \Rightarrow \dfrac{a^2}{1-r^2} = 150$

$\Rightarrow \quad \left(\dfrac{a}{1-r}\right)\left(\dfrac{a}{1+r}\right) = 150 \Rightarrow (15)\left(\dfrac{a}{1+r}\right) = 150 \Rightarrow \dfrac{a}{1+r} = 10 \qquad ...(ii)$

(i) & (ii) $\Rightarrow\ r = 1/5$

Now; $ar^2 + ar^4 + ar^6 + ...\infty = \dfrac{ar^2}{1-r^2} = \left(\dfrac{a}{1-r}\right)\left(\dfrac{r^2}{1+r}\right) = (15)\left(\dfrac{\left(\frac{1}{25}\right)}{1+\frac{1}{5}}\right) = \dfrac{1}{2}$

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Solutions of Archive - JEE Main & Advanced

Sequence and Series	Class - XI | Mathematics

JEE Main 2022

1.(2) Mass of acetic before adsorption

$$0.2 = \frac{w_1}{60} \times \frac{1000}{2000}$$

$$w_1 = 2.4g$$

Mass of acetic acid after adsorption

$$0.1 = \frac{w_2}{60} \times \frac{1000}{200}$$

$$w_2 = 1.2g$$

Mass of acetic acid adsorption

$$w_1 - w_2 = 2.4 - 1.2 = 1.2\,gram$$

Mass of acetic adsorption per gram of carbon $= \frac{1.2}{0.6} = 2$

2.(B) $a_1 + a_2 + a_3 + ... + a_n = 192$

$a_2 + a_4 + a_6 + ... = 120$ ...(i)

$\Rightarrow a_1 + a_3 + a_5 ..... = 192 - 120 = 72$ ...(ii)

Equation (i) – (ii)

$(a_2 - a_1) + (a_4 - a_3) + ... = 48$

$1 + 1 + 1 + ... + \frac{n}{2}$ times = 48

$\Rightarrow \frac{n}{2} = 48 \Rightarrow n = 96$

3.(98) $S = \frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \frac{65}{81} + ...100$ terms

$$= \left(\frac{3-2}{3}\right) + \left(\frac{3^2 - 2^2}{3^2}\right) + \left(\frac{3^3 - 2^3}{3^3}\right) + \left(\frac{3^4 - 2^4}{3^4}\right) + ...$$

$$= \left(1 + 1 + 1 + ...100\ terms\right) - \left(\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + ...100 terms\right)$$

$$= 100 - \frac{\frac{2}{3}\left(1 - \left(\frac{2}{3}\right)^{100}\right)}{1 - \frac{2}{3}}$$

$$S = 100 - 2 + \frac{2^{101}}{3^{100}}$$

$$\left[S\right] = \left[98 + \frac{2^{101}}{3^{100}}\right] = 98 + \left[\frac{2^{101}}{3^{100}}\right] = 98 + 0$$

$= 98$

4.(40) $a_2 + a_4 = 2a_3 + 1$

$a_1 r + a_1 r^3 = 2ar^2 + 1$ ... (i)

$3a_1 r + a_1 r^2 = 2a_1 r^3$

$a_1 r(3 + r - 2r^2) = 0$

$2r^2 - r - 3 = 0$

$2r^2 - 3r + 2r - 3 = 0$

$r(2r - 3) + 1(2r - 3) = 0$

$r = -1, r = \frac{3}{2}$

For $r = -1$

$-a_1 - a_1 = 2a_1 + 1$

$4a_1 = -1$

$a_1 = -\frac{1}{4} < 0$ but $a_1 > 0$

So, $r = -1$ (Not possible)

For $r = \frac{3}{2}$

$a_1 \times \frac{3}{2} + a_1 \times \frac{27}{8} = 2a_1 \times \frac{9}{4} + 1$

$\frac{12a_1 + 27a_1}{8} = \frac{36a_1}{8} + 1$

$39a_1 - 36a_1 = 8$

$a_1 = \frac{8}{3}$

$a_1 = \frac{8}{3}, a_2 = 4, a_3 = 6, a_4 = 9, a_5 = 27/2$

$a_2 + a_4 + 2a_5 = 4 + 9 + 27 = 40$

5.(C) $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + ...$

Considering infinite sequence,

$S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + ...$

$\frac{S}{7} = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + ...$

$\Rightarrow \quad \frac{6S}{7} = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \frac{10}{7^4} + ...$

$\Rightarrow \quad \frac{6S}{7^2} = \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \frac{8}{7^4} + ...$

$\frac{6S}{7}\left(1 - \frac{1}{7}\right) = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{6}{7^3} + ...$

$\Rightarrow \quad \frac{6^2 S}{7^2} = \frac{2}{1 - \frac{1}{7}} = \frac{2}{6} \times 7$

$\Rightarrow \quad S = \frac{2 \times 7^3}{6^3} \Rightarrow 4S = \frac{7^3}{3^3} = \left(\frac{7}{3}\right)^3$

6.(D) $a_1.a_2, a_3 ....A.P.; a_1 = 2; a_{10} = 3; d_1 = \frac{1}{9}$

$b_1, b_2, b_3, ...A.P.; b_1 = \frac{1}{2}; b_{10} = \frac{1}{3}; d_2 = \frac{-1}{54}$

[Using $a_1 b_1 = 1 = a_{10} b_{10}; d_1$ & d_2 are common differences respectively]

$$a_4.b_4 = \left(2 + 3d_1\right)\left(\frac{1}{2} + 3d_2\right) = \left(2 + \frac{1}{3}\right)\left(\frac{1}{2} - \frac{1}{18}\right) = \left(\frac{7}{3}\right)\left(\frac{8}{18}\right) = \frac{28}{27}$$

7.(C) $x = \sum_{n=0}^{\infty} a^n = 1 + a + a^2 + a^3 + ...\infty$

$$x = \frac{1}{1-a} \Rightarrow a = 1 - \frac{1}{x}$$

Similarly $y = \frac{1}{1-b} \Rightarrow b = 1 - \frac{1}{y}$

$$z = \frac{1}{1-c} \Rightarrow c = 1 - \frac{1}{z}$$

$\therefore a, b, c$ are in A.P.

$1 - \frac{1}{x}, 1 - \frac{1}{y}, 1 - \frac{1}{z}$ are in A.P.

$\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

8.(276) $S = \frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} +$ upto 10 terms

$$T_r = \frac{r}{1 + \left(2r^2\right)^2}$$

$$T_r = \frac{r}{1 + 4r^4}$$

$$T_r = \frac{r}{\left(2r^2 + 1\right)^2 - \left(2r\right)^2}$$

$$T_r = \frac{r}{\left(2r^2 - 2r + 1\right)\left(2r^2 + 2r + 1\right)}$$

$$T_r = \frac{1}{4} \frac{\left(2r^2 + 2r + 1\right) - \left(2r^2 - 2r + 1\right)}{\left(2r^2 - 2r + 1\right)\left(2r^2 + 2r + 1\right)}$$

$$S_{10} = \sum_{r=1}^{10} T_r = 276$$

9.(C)

$$\underbrace{a, A_1, A_2, A_3, ... A_n, 100}_{(n+2) \text{ terms}}$$

$$\therefore d = \frac{100 - a}{n + 1}$$

$$\frac{A_n}{A_1} = \frac{a + nd}{a + d} = \frac{a + n\left(\frac{100 - a}{n + 1}\right)}{a + \frac{100 - a}{n + 1}} = 7$$

$\Rightarrow \dfrac{an+a+100n-an}{an+a+100-a}=7$

$=\dfrac{33-n+100n}{n(33-n)+100}=7$

$\Rightarrow 33+99n=7\left(-n^2+33n+100\right)$

$\Rightarrow 7n^2-132n-667$

On solving $n=23$.

10.(41651) $S_n=\dfrac{n^2}{1-\dfrac{1}{(n+1)^2}}=\dfrac{n(n+1)^2}{(n+2)}$

$S_n=\dfrac{n\left(n^2+2n+1\right)}{(n+2)}$

$S_n=\dfrac{n\left[n(n+2)+1\right]}{(n+2)}$

$S_n=n\left[n+\dfrac{1}{n+2}\right]$

$S_n=n^2+\dfrac{n+2-2}{(n+2)}$

$S_n=n^2+1-\dfrac{2}{(n+2)}$

Now $\dfrac{1}{26}+\sum_{n=1}^{50}\left[\left(n^2-n\right)-2\left(\dfrac{1}{n+2}-\dfrac{1}{n+1}\right)\right]=\dfrac{1}{26}+\left[\dfrac{50\times51\times101}{6}-\dfrac{50\times51}{2}-2\left(\dfrac{1}{52}-\dfrac{1}{2}\right)\right]$

= 41651

11.(C)

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
$\dfrac{a}{r^3}$	$\dfrac{a}{r^2}$	$\dfrac{a}{r}$	a	ar	ar^2	ar^3	ar^4	ar^5	ar^6

$\dfrac{a}{r^3}.\dfrac{a}{r}.ar.ar^3=\dfrac{1}{1256}$

$a^4=\dfrac{1}{1296}$

$a=\pm\dfrac{1}{6}\rightarrow neglect\ -\dfrac{1}{6}$

$\dfrac{a}{r^2}+a=\dfrac{7}{36}$

$a\left(\dfrac{1}{r^2}+1\right)=\dfrac{7}{36}\Rightarrow\dfrac{1}{r^2}+1=\dfrac{7}{6}\Rightarrow\dfrac{1}{r^2}=\dfrac{1}{6}\Rightarrow r=\pm\sqrt{6},r=+\sqrt{6}$

Neglect $r=-\sqrt{6}$

Now $A_6+A_8+A_{10}$

Now $A_6+A_8+A_{10}$

$ar^2+ar^4+ar^6$

$a\left(r^2+r^4+r^6\right)$

$\dfrac{1}{6}(6+36+216)$

$(1+6+36)=43$

12.(C) Denominators of given series are in G.P.

Denominators of numerators are in A.P.

Let N_k denote the kth numerator

$N_{k+1} - N_k = 3k + 1$

$\Rightarrow \quad N_2 - N_1 = 4$

$N_3 - N_2 = 7$

$N_r - N_{r-1} = 3r - 2$

$\Rightarrow \quad N_r - N_1 = \frac{(r-1)}{2}(3r+2)$

$\Rightarrow \quad N_r = \frac{(r-1)(3r+2)}{2} + 1 = \frac{3r^2 - r}{2}$

Also: $D_r = r^{th}$ denominator $= \left(\frac{1}{6}\right)^{r-1}$

$\Rightarrow \quad S = \sum_{r=0}^{\infty} \frac{(r+1)(3r+2)}{6^r}$

$\Rightarrow \quad \frac{S}{6} = \frac{1}{6} + \frac{5}{6^2} + \frac{12}{6^3} .....$

$S - \frac{5S}{6} = 1 + \frac{5-1}{6} + \frac{12-5}{6^2} + \frac{22-12}{6^3} .....$

$\frac{5S}{6} = 1 + \frac{4}{6} + \frac{7}{6^2} + \frac{10}{6^3} .....$

$\frac{5S}{36} = \frac{1}{6} + \frac{4}{6^2} + \frac{7}{6^3}$

$\frac{5S}{6} - \frac{5S}{36} = 1 + \frac{3}{6} + \frac{3}{6^2} + \frac{3}{6^3} .....$

$\frac{25S}{36} = 1 + 3\left(\frac{1/6}{5/6}\right) = \frac{8}{5}$

$S = \frac{288}{125}$

13.(2223) First AP $3, 6, 9, 12, \ldots\ldots\ldots\ldots, 234$

Second AP $5, 9, 13, 17 \ldots\ldots\ldots\ldots, 237$

Common terms of AP are 9, 21, 33, ...

Let no. of common terms be n

Then $9 + (n-1)12 \le 234$

$\Rightarrow \quad 12n - 3 \le 234$

$\Rightarrow \quad 12n \le 237$

$\Rightarrow \quad n \le 19.75$

$\Rightarrow \quad \boxed{n = 19}$

$\therefore$ Sum of common terms

$= 9 + 21 + 33 + 45 + \ldots\ldots$ upto 19 terms

$= \frac{19}{2}[2 \times 9 + (19-1)12]$

$= \frac{19}{2}[18 + 216] = 2223$

14.(B) $a_{n+2} - a_{n+1} - a_n + 1$

$n = 0 \quad a_2 - a_1 = a_1 - a_0 + 1$

$n = 1 \quad a_3 - a_2 = a_2 - a_1 + 1$

$n = 2 \quad a_4 - a_3 = a_3 - a_2 + 1$

.....

.....

$a_{n+2} - a_{n+1} = a_{n+1} - a_n + 1$

Adding $a_{n+2} - a_1 = a_{n+1} - a_0 + (n+1)$

$\Rightarrow \quad a_{n+2} - a_{n+1} = n + 1$

$n = 0 \quad a_2 - a_1 = 1$

$n = 1 \quad a_3 - a_2 = 2$

$n = 2 \quad a_4 - a_3 = 3$

.....

.....

$a_{n+2} - a_{n+1} = n + 1 \Rightarrow a_{n+2} = \dfrac{(n+1)(n+2)}{2}$

$\Rightarrow a_n = \dfrac{(n-1)n}{2} \Rightarrow \dfrac{1}{2}\sum_{n=2}^{\infty} \dfrac{(n-1)n}{7^n}$

$\dfrac{1}{2}\left(\dfrac{1\cdot 2}{7^2} + \dfrac{2\cdot 3}{7^3} + \dfrac{3\cdot 4}{7^4} + \ldots\right)$

$S_\infty = \dfrac{1\cdot 2}{7^2} + \dfrac{2\cdot 3}{7^3} + \dfrac{3\cdot 4}{7^4} + \dfrac{4\cdot 5}{7^5} + \ldots$

$\dfrac{1}{7}S_n = \dfrac{1\cdot 2}{7^3} + \dfrac{2\cdot 3}{7^4} + \dfrac{3\cdot 4}{7^5} + \ldots$

$\dfrac{6}{7}S_\infty = \dfrac{1\cdot 2}{7^2} + \dfrac{2^2}{7^3} + \dfrac{2\cdot 3}{7^4} + \dfrac{2\cdot 4}{7^5} + \ldots..$

$= \dfrac{2}{7^2}\left(1 + \dfrac{2}{7} + \dfrac{3}{7^2} + \dfrac{4}{7^3} + \ldots\right)$

$= \dfrac{2}{7^2}\left(\dfrac{1}{1-\frac{1}{7}} + \dfrac{1\times\frac{1}{7}}{\left(1-\frac{1}{7}\right)^2}\right)$

$= \dfrac{2}{49}\left(\dfrac{7}{6} + \dfrac{7}{36}\right) = \dfrac{1}{18}$

$S_\infty = \dfrac{1}{18} \times \dfrac{7}{6} = \dfrac{7}{108}$

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Solutions of Archive - JEE Main & Advanced

Complex Numbers	Class - XI | Mathematics

JEE Main 2021

1.(10) $Z = x + iy$

$x + iy + \alpha\sqrt{(x-1)^2 + y^2} + 2i = 0$

$y + 2 = 0 \; ; \; x + \alpha\sqrt{(x-1)^2 + y^2} = 0$

$y = -2 \; ; \; x^2 = \alpha^2(x^2 - 2x + 1 + 4)$

$\alpha^2 = \dfrac{x^2}{x^2 - 2x + 5}$

$\alpha^2 \in [0, 5/4] \Rightarrow \alpha \in \left[-\dfrac{\sqrt{5}}{2}, \dfrac{\sqrt{5}}{2}\right] ; P = -\dfrac{\sqrt{5}}{2}, \; q = \dfrac{\sqrt{5}}{2} \Rightarrow 4(p^2 + q^2) = 4 \times (5/4 + 5/4) = 10$

2.(310) $-1 + i\sqrt{3} = 2\omega$

$(-1 + i\sqrt{3})^{21} = 2^{21}$

$(1 + i\sqrt{3}) = -2\omega^2$

$(1 + i\sqrt{3})^{21} = -2^{21}$

$(1 - i)^{24} = \left(\sqrt{2} \cdot e^{-i\pi/4}\right)^{24} = 2^{12} \cdot e^{-i6\pi} = 2^{12}$

$(1 + i)^{24} = 2^{12} \quad \therefore \quad k = 0$

$\sum_{j=0}^{5} (j+5)^2 - \sum_{j=0}^{5} (j+5) = (5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2) - (5 + 6 + 7 + 8 + 9 + 10) = 310$

3.(D) Putting $z = x + yi$

$\dfrac{(2-i)}{2+i} = \dfrac{\bar{z}}{z} \Rightarrow 2y = x$ ...(i) and $(2+i)(x+yi) + (i-2)(x-yi) - 4i = 0 \Rightarrow x + 2y - 2 = 0$ ...(ii)

On solving (i) and (ii) $P\left(1, \dfrac{1}{2}\right)$

Now line $iz + \bar{z} + 1 + i = 0 \Rightarrow x - y + 1 = 0$ which is at a distance $\dfrac{3}{2\sqrt{2}}$ from $\left(1, \dfrac{1}{2}\right)$

4.(B) $\because \; \alpha, \beta \in R$

Given eq^n : $z^2+\alpha z+\beta=0$

If one root is $1-2i$, then other root is $1+2i$.

$\therefore$ Sum of roots $=-\alpha \quad \Rightarrow \quad (1+2i)+(1-2i)=-\alpha \Rightarrow \alpha=-2$

Also product of roots $=\beta$

$(1+2i)(1-2i)=\beta \quad \Rightarrow \quad \beta=1+4=5 \quad \therefore \quad \alpha-\beta=-2-5=-7$

5.(3) $x^3-2x^2+2x-1=0$; $(x-1)(x^2-x+1)=0$; $x=1, \quad x=\dfrac{1\pm\sqrt{3}i}{2}$

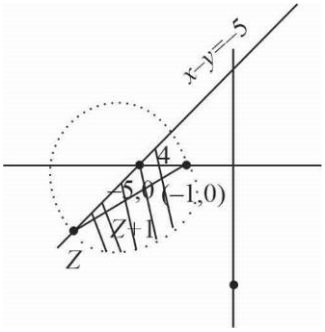
$x=-\omega,-\omega^2$; $(1)^{162}+(-\omega)^{162}+(-\omega^2)^{162}=1+\omega^{162}+\omega^{324}=1+1+1=3$

6.(48) $|z+5|\le 4$

$z(1+i)+\bar{z}(1-i)\ge -10$

Max. value of $|z+1|^2$ is $\alpha+\beta\sqrt{2}$

$(\alpha+\beta)=?$



$z+\bar{z}+i(z-\bar{z})\ge -10$

$x+iy+x-iy+i(x+iy-x+iy)\ge -10$

$2x+2yi^2\ge -10$; $2x-2y\ge -10$

$x-y\ge -5$ ...(i)

$|z+5|\le 4$ ...(ii)

$(x+5)^2+y^2=16$

$x=y-5$

$(y-5+5)^2+y^2=16$; $2y^2=16\Rightarrow y^2=8$; $y=-2\sqrt{2}$

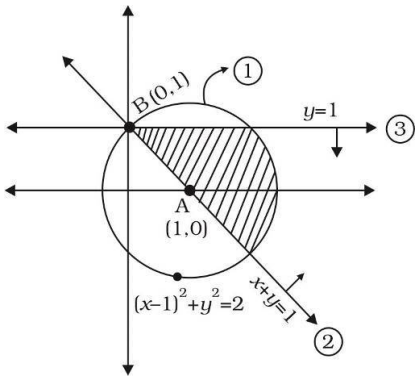
$x=-2\sqrt{2}-5$; $z=-2\sqrt{2}-5-2\sqrt{2}i$

$z+1=-2\sqrt{2}-4-i2\sqrt{2}$; $|z+1|^2=(-2\sqrt{2}-4)^2+(2\sqrt{2})^2=32+16\sqrt{2}\Rightarrow \alpha+\beta=48$

7.(D) $S_1=\{Z\in C:|z-1|\le\sqrt{2}\}$ ...(i)

$S_2=\{Z\in C:\text{Re}((1-i)z)\ge 1\}$ ...(ii)

$S_2=\{Z\in C:\text{Im}(z)\le 1\}$ ...(iii)



S_1 represents all point lying on a circle and interior to the circle having centre $A(1+0i)$ with radius $\sqrt{2}$.

Let $z=x+iy\Rightarrow \text{Re}[(1-i)z]\ge 1\Rightarrow x+y\ge 1$

$S_3:\text{Im}(x+iy)\le 1\Rightarrow y\le 1$

Hence, shaded portion represents $S_1\cap S_2\cap S_3$.

8.(A) $a|z|^2+\overline{\bar{\alpha}z+\alpha\bar{z}}+d=0$ or $a|z|^2+\alpha\bar{z}+\bar{\alpha}z+d=0$

Comparing this equation w.r.t. $|z - z_0| = r$

$\Rightarrow \quad (z - z_0)(\bar{z}_0 - \bar{z}_0) = r^2 \quad$ or $\quad |z|^2 - \bar{z}z_0 - \bar{z}_0 z + |z_0|^2 - r^2 = 0$

or $\quad \frac{a}{1} = \frac{\alpha}{-z_0} = \frac{d}{|z_0|^2 - r^2} \quad \Rightarrow \; a \in R$

or $\quad |z_0|^2 - r^2 = \frac{d}{a} \quad$ or $\quad |z_0|^2 = \frac{|z^2|}{a^2}$

$\frac{|z^2|}{a^2} - r^2 = \frac{d}{a} \quad \Rightarrow \quad r^2 = \frac{|\alpha|^2}{a^2} - \frac{d}{a} = \frac{|\alpha^2| - ad}{a^2}$

or $\quad |\alpha|^2 - ad \geq 0$

9.(6) if z_1, z_2 and origin form an equilateral triangle, then either

$z_2 = z_1 e^{i\pi/3} \quad$ or $\quad z_1 = z_2 e^{i\pi/3} \Rightarrow \quad z_2^3 + z_1^3 = 0 \Rightarrow \quad z_2^2 - z_1 z_2 + z_1^2 = 0$

$\Rightarrow \quad (z_1 + z_2)^2 = 3z_1 z_2$

$z_1 + z_2 = -a \; ; \; z_1 z_2 = 12 \Rightarrow \quad a^2 = 36 \Rightarrow \quad a = \pm 6 \quad \Rightarrow \quad |a| = 6$

10.(D) $\left|5\sqrt{7} + 9i\right| = \sqrt{(25 \times 7) + 81} = 16$

L.H.S. $= e^{\left(\frac{(|z|+3)(|z|-1)}{|z|+1} \log_e 2\right)} = e^{\log_e 2^{\left(\frac{(|z|+3)(|z|-1)}{|z|+1}\right)}} = 2^{\left(\frac{(|z|+3)(|z|-1)}{|z|+1}\right)}$

R.H.S. $= \log_{\sqrt{2}} 16 = 2\log_2 16 = 8 = 2^3$

To solve $\quad \Rightarrow \left(\frac{(|z|+3)(|z|-1)}{|z|+1}\right) \geq 3$

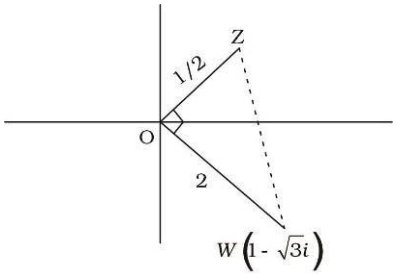
Multiply by $|z| + 1$ (always +ve) $\quad \Rightarrow (|z|+3)(|z|-1) \geq 3|z| + 3$

Let $|z| = t > 0 \quad \Rightarrow t^2 - t - 6 \geq 0 \Rightarrow (t-3)(t+2) \geq 0 \quad \Rightarrow t \in (-\infty, -2] \cup [3, \infty)$

As $t \geq 0 \quad \Rightarrow |z| = 3$, minimum value.

11.(C) Clearly the area of triangle is $\frac{1}{2}|z||z\,\hat{i}| = \frac{1}{2}|z|^2$

12.(A) Area of $\Delta WOZ = \frac{1}{2} \times 2 \times \frac{1}{2} = \frac{1}{2}$



13.(0) $P(x)$ is divisible by $x^2 + x + 1 = (x - \omega)(x - \omega^2)$

Where ω & ω^2 are non-real complex cube root of unity

$P(\omega) = 0 \quad \Rightarrow f(1) + \omega g(1) = 0 \quad \ldots(i)$

$P(\omega^2) = 0$

$f(1) + \omega^2 g(1) = 0 \quad \ldots(ii)$

From (i) & (ii)

$f(1) = g(1) = 0 \quad \therefore P(1) = f(1) + g(1) = 0$

14.(C) $\because \quad \log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \le 2 \quad \Rightarrow \quad \frac{|z|+11}{(|z|-1)^2} \ge \frac{1}{2}$

$\Rightarrow \quad |z|^2 - 4|z| - 21 \le 0 \quad \Rightarrow \quad |z| \le 7 \quad \therefore$ Largest value of $|z| = 7$

15.(4) $\because \quad |z+i| = |z-3i| \quad \Rightarrow$ z lies on the line $y = 1 \quad \therefore \quad z = x + i$

Now, $\omega = z\bar{z} - 2z + 2$

$\omega = (x+i)(x-i) - 2(x+i) + 2$

$\omega = (x^2 - 2x + 3) - 2i$

$\text{Re}(\omega) = x^2 - 2x + 3$

For min $\text{Re}(\omega)$, $x = 1 \quad \Rightarrow \quad \omega = 2 - 2i \quad \therefore$ Minimum value of

$n, (n \in N), \omega^n = (2(1-i))^n$

Is real is $n = 4$

16.(A) $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right) = \arg\left(\frac{1-2i}{1+3i}\right) = \arg(1-2i)(1-3i) = \arg(-5-5i) = -\frac{3\pi}{4}$

17.(C) $z = \frac{1}{1-\cos\theta+2i\sin\theta} \times \frac{1-\cos\theta-2i\sin\theta}{1-\cos\theta-2i\sin\theta}$

$= \frac{1-\cos\theta-2i\sin\theta}{(1-\cos\theta)^2+(2\sin\theta)^2}$

$R(z) = \frac{1-\cos\theta}{(1-\cos\theta)^2} + 4\sin^2\theta = \frac{1}{5} \quad \Rightarrow \quad 5 - 5\cos = 1 + \cos^2\theta - 2\cos\theta + 4 - 4\cos^2\theta$

$\Rightarrow \quad 3\cos^2\theta - 3\cos\theta = 0$

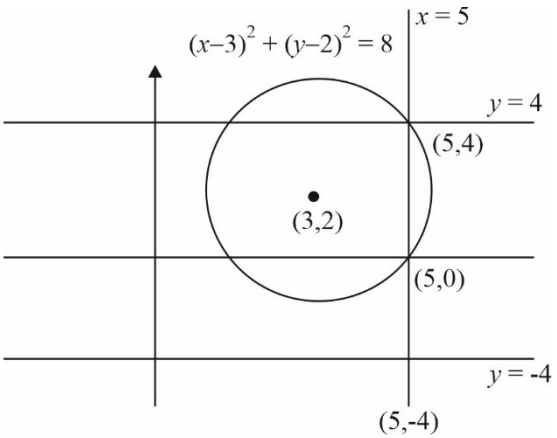
$\cos\theta(\cos\theta - 1) = 0 \quad \theta = 0, \frac{\pi}{2} \quad \theta c - (0, \pi)$

$\int_0^{\pi/2} \sin x \, dx = \left[-\cos x\right]_0^{\pi/2} = 1$

18.(B) $S_1 = \left\{z \in C \mid |x-3-2i|^2 = 8\right\}$ represents a circle centered at (3,2) and radius $= 2\sqrt{2}$

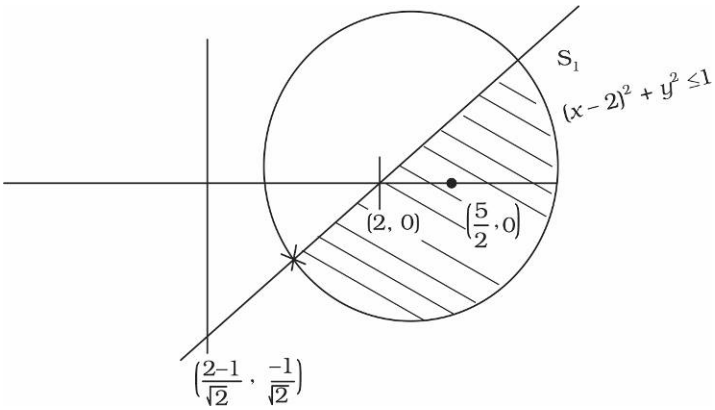
$S_2 = \{z \in C \mid \text{Re}(z) \ge 5\} : x \ge 5$

$S_3 = \left\{z \in C \mid |z - \bar{z}| \ge 8\right\} : y \ge 4$



Number of solution = 1

19.(B) $S_2 = z(1+i) + \bar{z}(1-i) \geq 4$



$x - y \geq 2$

$S_1 \; n S_2$

$$d = \left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$d = \frac{1}{4} + \frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{1}{2}$$

$$d = \frac{5}{4} + \frac{1}{\sqrt{2}}$$

20.(1) $$z = \frac{(3 + 2i\cos\theta)(1 + 3i\cos\theta)}{(1 - 3i\cos\theta)(1 + 3i\cos\theta)} = \frac{3 - 6\cos^2\theta + 11i\cos 1}{1 + 9\cos^2\theta}$$

Real (2) = 0 $\Rightarrow$ $\cos^2\theta = \frac{1}{2}$ $\Rightarrow$ $\theta = 45°$

$$\sin^2(135°) + \cos^2(45°) = \frac{1}{2} + \frac{1}{2} = 1$$

21.(A) Given $\frac{z-i}{z+2i} \in R$ $\Rightarrow$ $\arg\left(\frac{z-i}{z+2i}\right) = 0 \text{ or } \pi$ $\Rightarrow$ $i, -2i$ and z are collinear

$\Rightarrow$ z lies on y-axis

22.(6) $\dfrac{(2i)^n}{(1-i)^{n-2}}\times\left(\dfrac{1+i}{1+i}\right)^{n-2}=\dfrac{(2i)^n(1+i)^{n-2}}{(1-i^2)^{n-2}}=\dfrac{(2i)^n(1+i)^{n-2}}{2^{n-2}}=\dfrac{2^n\cdot i^n(1+i)^{n-2}}{2^{n-2}}=4\cdot i^n(1+i)^{n-2}$

$n=2$ $\quad -4$

$n=4$ $\quad =4(2i)$

$n=6$ $\quad 4\cdot i^6\cdot(1+i)^4=4i^2(2i)^2=-4(-4)=16$

23.(A) $(\sqrt{3}+i)^{100}=2^{99}(p+iq)$

$(\sqrt{3}+i)=2e^{i\pi/6}$

$\Rightarrow \quad (\sqrt{3}+i)^{100}=2^{100}e^{i\left(100\frac{\pi}{6}\right)}=2^{99}\left(2e^{i\left(50\frac{\pi}{3}\right)}\right)=2^{99}(-1+\sqrt{3}i)$

$\Rightarrow \quad p=-1$ and $q=\sqrt{3}$

Equation is $x^2-(\sqrt{3}-1)x-\sqrt{3}=0$

However it's not given that $p,q\in\mathbb{R}$ and so there can be many such equations.

24.(13) $z=\dfrac{1-i\sqrt{3}}{2}=-\left(\dfrac{-1+i\sqrt{3}}{2}\right)=-\omega$

Now, $S=21+\left(z+\dfrac{1}{z}\right)^3+\left(z^2+\dfrac{1}{z^2}\right)^3+\left(z^3+\dfrac{1}{z^3}\right)^3+\ldots+\left(z^{21}+\dfrac{1}{z^{21}}\right)^3$

$=21+\left\{\left[-\left(\omega+\dfrac{1}{\omega}\right)^3\right]+\left(\omega^2+\dfrac{1}{\omega^2}\right)^3-\left(\omega^3+\dfrac{1}{\omega^3}\right)^3+\left(\omega^4+\dfrac{1}{\omega^4}\right)^3-\left(\omega^5+\dfrac{1}{\omega^5}\right)^3+\left(\omega^6+\dfrac{1}{\omega^6}\right)^3+\ldots\ldots\right.$

$\left.+\left[-\left(\omega^{13}+\dfrac{1}{\omega^{13}}\right)^3\right]+\left(\omega^{14}+\dfrac{1}{\omega^{14}}\right)^3-\left(\omega^{15}+\dfrac{1}{\omega^{15}}\right)^3+\left(\omega^{16}+\dfrac{1}{\omega^{16}}\right)^3-\left(\omega^{17}+\dfrac{1}{\omega^{17}}\right)^3+\left(\omega^{18}+\dfrac{1}{\omega^{18}}\right)^3\right\}$

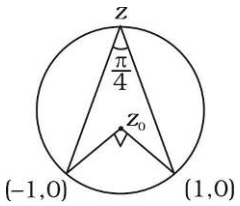
$+\left[-\left(\omega^{19}+\dfrac{1}{\omega^{19}}\right)^3\right]+\left(\omega^{20}+\dfrac{1}{\omega^{20}}\right)^3-\left(\omega^{21}+\dfrac{1}{\omega^{21}}\right)^3$

Clearly, all the terms present in curly bracket cancels each other.

Hence, $S=21+\{0\}+\left[-\left(\omega+\dfrac{1}{\omega}\right)^3\right]+\left(\omega^2+\dfrac{1}{\omega^2}\right)^3-(2^3)$

$=21-\left(\dfrac{\omega^2+1}{\omega}\right)^3+\left(\dfrac{\omega^4+1}{\omega^2}\right)^3-8=21-\left(-\dfrac{\omega}{\omega}\right)^3+\left(-\dfrac{\omega^2}{\omega^2}\right)^3-8=21+1-1-8=13$

25.(B) $\arg\left(\dfrac{z-1}{z+1}\right)=\dfrac{\pi}{4}$

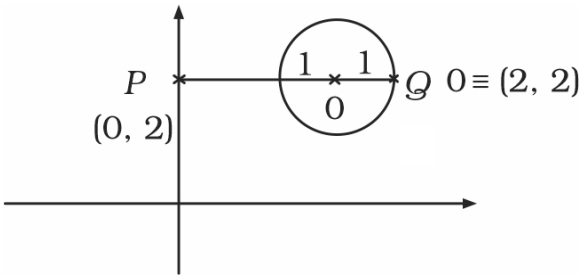


$\Rightarrow \quad \arg\left(\frac{z_0-1}{z_0+1}\right)=\frac{\pi}{2} \quad \Rightarrow \quad \frac{z_0-1}{z_0+1}=\left|\frac{z_0-1}{z_0+1}\right|e^{\frac{i\pi}{2}}$

$\Rightarrow \quad \frac{z_0-1}{z_0+1}=i \quad \{\because |z_0-1|=|z_0+1| \text{ and } e^{\frac{i\pi}{2}}=i\}$

Hence, Centre : $(0,1)$ & Radius = $\sqrt{2}$

26.(5)



$|3iz+6|=3|z-2i|$

Maximum value of $3|z-2i|$ woule be attained at Q (3, 2)

$Q(z)=3+2i=a+ib$

27.(98) Let $z=x+iy$

$\arg\left(\frac{x-2+iy}{x+2+iy}\right)=\frac{\pi}{4}$

$\arg(x-2+iy)-\arg(x+2+iy)=\frac{\pi}{4}$

$\tan^{-1}\left(\frac{y}{x-2}\right)-\tan^{-1}\left(\frac{y}{x+2}\right)=\frac{\pi}{4}$

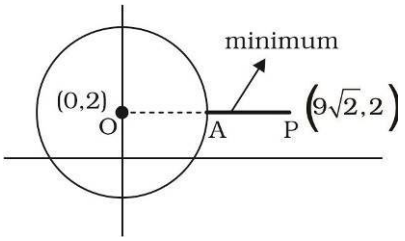
$\frac{\frac{y}{x-2}-\frac{y}{x+2}}{1+\left(\frac{y}{x-2}\right)\cdot\left(\frac{y}{x+2}\right)}=\tan\frac{\pi}{4}=1$

$\frac{xy+2y-xy+2y}{x^2-4+y^2}=1$

$4y=x^2-4+y^2$

$x^2+y^2-4y-4=0$

Locus is a circle with centre (0,2) & radius $=2\sqrt{2}$



min. value $= (AP)^2 = (OP - OA)^2 = \left(9\sqrt{2} - 2\sqrt{2}\right)^2 = \left(7\sqrt{2}\right)^2 = 98$

28.(B) $a_r = e^{i2\pi r/9}$; $r = 1,2,3....$

$a_1, a_2, a_3, ...$ are in G.P.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} a_1 & a_1^2 & a_1^3 \\ a_1^4 & a_1^5 & a_1^6 \\ a_1^7 & a_1^8 & a_1^9 \end{vmatrix} = a_1 a_1^4 a_1^7 \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \end{vmatrix} = 0 \qquad \text{and } a_1 a_9 - a_3 a_7 = a_1^{10} - a_1^{10} = 0$$

29.(6) $Arg\left(z_1 - z_2\right) = \dfrac{\pi}{4} \Rightarrow \dfrac{y_1 - y_2}{x_1 - x_2} = \tan\dfrac{\pi}{4}$

$\Rightarrow y_1 - y_2 = x_1 - x_2$

and $\left|z - 3\right| = \text{Re}\left(z\right)$

$\left(x - 3\right)^2 + y^2 = x^2$

$\Rightarrow y^2 = 6x - 9$

$y_1^2 - y_2^2 = \left(6x_1 - 9\right)\left(6x_2 - 9\right)$

$\left(y_1 - y_2\right)\left(y_1 + y_2\right) = 6\left(x_1 - x_2\right) \Rightarrow y_1 + y_2 = 6$

30.(C) Let $z = x + iy$

$z^2 + 3\bar{z} = 0 \Rightarrow x^2 - y^2 + i2xy + 3x - i3y = 0 \Rightarrow x^2 - y^2 + 3x = 0$

$\Rightarrow \qquad x^2 - y^2 + 3x = 0$ & $\left\{y = 0\left(OR\right)x = 3/2\right\}$

Case I: $y = 0 \Rightarrow x^2 - 0^2 + 3x = 0 \Rightarrow x = 0, -3$

i.e. $x = 0, y = 0$; $x = -3, y = 0 \Rightarrow z = 0, -3$

Case II: $x = 3/2 \Rightarrow \dfrac{9}{4} - y^2 + 3\left(\dfrac{3}{2}\right) = 0 \Rightarrow y = \pm\dfrac{3\sqrt{3}}{2} \Rightarrow z = \dfrac{3}{2} \pm i\dfrac{3\sqrt{3}}{2}$

Hence, $k = 4$

Now; $\displaystyle\sum_{k=0}^{\infty}\frac{1}{n^k} = \sum_{k=0}^{\infty}\frac{1}{4^k} = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$

JEE Advanced 2021

1.(C) $z_1 = e^{i\theta_1}$

$z_k = z_{k-1}e^{i\theta_k}$

$z_2 = e^{i(\theta_1+\theta_2)}$

$z_3 = e^{i(\theta_1+\theta_2+\theta_3)}$

.

.

$z_{10} = e^{i(\theta_1+\theta_2.....\theta_{10})} = e^{i2\pi} = 1$

$|z_2 - z_1| = \sqrt{(\cos(\theta_1+\theta_2)-\cos\theta_1)^2 + (\sin(\theta_1+\theta_2)-\sin\theta_1)^2}$

$= \sqrt{2-2\cos\theta_2} = \left|2\sin\frac{\theta_2}{2}\right|$

$|z_3 - z_2| = \left|2\sin\frac{\theta_3}{2}\right|$

.

,

$(z_{10} - z_1) = \left|2\sin\frac{\theta_1}{2}\right|$

$2\left(\left|\sin\frac{\theta_2}{2}\right| + \left|\sin\frac{\theta_3}{2}\right| + ...\left|\sin\frac{\theta_{10}}{2}\right| + \left|\sin\frac{\theta_1}{2}\right|\right)$

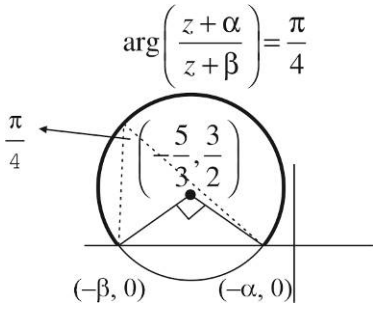
$\sin\theta < \theta \ \ \forall \ \ \theta \in (0, 2\pi)$

$\leq \left[2\left(\frac{\theta_2}{2} + \frac{\theta_3}{2}.....\frac{\theta_{10}}{2} + \frac{\theta_1}{2}\right)2\pi\right]$

Like wise for *q* statement also

$2|\sin\theta_2| + 2|\sin\theta_3| + ...2|\sin\theta_{10}| + 2|\sin\theta_1|$

2.(BD)



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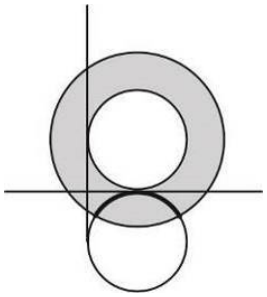
1.(B) Student may get confused as coeffcient of x in the given quadratic equation is missing.

2.(D) $A = \{Z \in C ; 1 \le |Z-(1+i)| \le 2\}$ & $B = \{Z \in A : |Z-(1-i)| = 1\}$



3.(C) $\left|-3\sqrt{2} + p\sqrt{2}i\right| = 5\sqrt{2}$

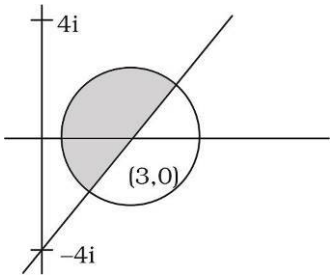
$18 + 2p^2 = 50 \Rightarrow p^2 = 16p \Rightarrow \pm 4$

4.(80) Here $|z-3| \le 1$

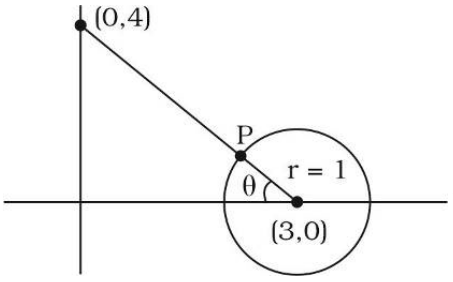
$\Rightarrow (x-3)^2 + y^2 \le 1$

and $z(4+3i) + \bar{z}(4-3i) \le 24$

$\Rightarrow 4x - 3y \le 12$



$\tan\theta = \frac{4}{3}$



$\therefore$ Coordinate of $P = (3-\cos\theta, \sin\theta) = \left(3-\frac{3}{5}, \frac{4}{5}\right)$

$\therefore\ \alpha + i\beta = \frac{12}{5} + \frac{4}{5}i$

$\therefore\ 25(\alpha+\beta) = 80$

5.(C) $\bar{Z}_1 = i\bar{Z}_2$

$\Rightarrow \overline{(\bar{Z}_1)} = \overline{(i\bar{Z}_2)}$

$\Rightarrow Z_1 = -iZ_2$ ...(i)

Given,

$\arg\left(\frac{Z_1}{\bar{Z}_2}\right) = \pi$

$\Rightarrow \arg\left(\frac{-iZ_2}{\bar{Z}_2}\right) = \pi$ *from (i)*

$\Rightarrow \arg\left(-iZ_2\right) - \arg\left(\bar{Z}_2\right) = \pi$

$\Rightarrow \arg\left(-iZ_2\right) + \arg(Z_2) = \pi$

Let $\arg(Z) = \theta$

$-\frac{\pi}{2} + \theta + \theta = \pi$

$\Rightarrow 2\theta = \frac{3\pi}{2} \Rightarrow \theta = \frac{3\pi}{4}$

$\arg(Z_2) = \frac{3\pi}{4}$ *and* $\arg(Z_1) = \frac{\pi}{4}$

6.(B) $z_1\left(3+4i\right) \equiv A_1\left(3,4\right)$

$z_2\left(4+3i\right) \equiv A_2\left(4,3\right)$

$z_3\left(5i\right) \equiv A_3\left(0,5\right)$

$z\left(\neq z\right) \equiv A\left(x,y\right)$

$AA_1 \perp A_2A_3 \Rightarrow \left(\frac{y-4}{x-3}\right)\left(\frac{5-3}{0-4}\right) = -1 \Rightarrow 2x - 2 = y$ .... (i)

Equation of circle : $x^2 + y^2 = 25$ ...(ii)

(i) & (ii) $\Rightarrow$ $x = 3, \frac{-7}{5}$

$y = 4, -24/5$

$\therefore$ $z \equiv \left(\frac{-7}{5} - \frac{24}{5}i\right) \Rightarrow \arg\left(z\right) = \tan^{-1}\left|\frac{-24/5}{-7/5}\right| - \pi$

$= \tan^{-1}\left(24/7\right) - \pi$

7.(2) As given $z^2 + z + 1 = 0$

$\therefore$ $z = \frac{-1 \pm \sqrt{3}i}{2} = \omega, \omega^2$

Now, $\sum_{n=1}^{15}\left(z^n + (-1)^n \frac{1}{z^n}\right)^2$

$= \sum_{n=1}^{15}\left(z^{2n} + \frac{1}{z^{2n}} + 2(-1)^n\right) = \sum_{n=1}^{15}\left[z^{2n} + z^n + 1 + 2(-1)^n - 1\right]$

$$= \sum_{n=1}^{15}(1+z^n+z^{2n}) + \sum_{n=1}^{15}(2(-1)^n - 1)$$

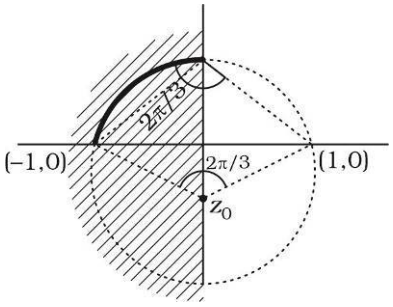
$$= (3\times 5) + (2\times 7 - (2)(8) - 15)$$

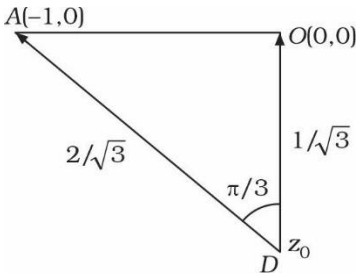
$$= 15 + (-17) = -2$$

$$\therefore \quad \left|\sum_{n=1}^{15}\left(z^n + (-1)^n \frac{1}{z^n}\right)^2\right| = |-2| = 2$$

8.(B) As given $A = \left\{z \in C : \left|\frac{z+1}{z-1}\right| < 1\right\}$ and $B = z \in C : \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3}$

$$\Rightarrow \quad |z+1| < |z-1|$$



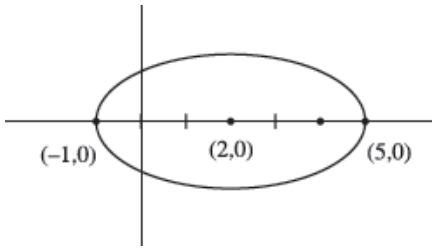


Using rotation theorem $\frac{0-z_0}{(1/\sqrt{3})} \times e^{i\pi/3} = \frac{(-1-z_0)}{2/\sqrt{3}}$

After solving we get $z_0 = -(1/\sqrt{3})i$

9.(C) $(x-4)^2 + (y-3)^2 = 4$

$$E : \frac{(x-2)^2}{9} + \frac{y^2}{5} = 1$$



Lower Extermity of vertical diameter of circle $\rightarrow$ (4, 1)

Put in ellipse $\Rightarrow$ $\frac{(4-2)^2}{9} + \frac{1}{5} - 1$

$$= \frac{4}{9} + \frac{1}{5} - 1$$

$$= \frac{29}{45} - 1 < 0$$

Two solutions

10.(B) $a = \alpha - i\beta; \alpha \in R; \beta \in R$

$4ix + (1+i)y = 0$ and $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + \bar{a}y = 0$

$$\begin{vmatrix} 4i & 1+i \\ 8e^{i2\pi/3} & \bar{a} \end{vmatrix} = 0$$

$\Rightarrow \quad 4i\bar{a} - (1+i)8e^{i2\pi/3} = 0$

$\Rightarrow \quad 4i(\alpha + i\beta) - 8(1+i)\left(\frac{-1+i\sqrt{3}}{2}\right) = 0$

$\Rightarrow \quad i\alpha - \beta + 1 + \sqrt{3} + i(1-\sqrt{3}) = 0$

$\Rightarrow \quad i\alpha - \beta + 1 + \sqrt{3} + i(1-\sqrt{3}) = 0$

$\Rightarrow \quad \beta = \sqrt{3} + 1$

$\alpha = \sqrt{3} - 1$

So, $\frac{\alpha}{\beta} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3}$

11.(A) Let $Z = x + iy$

According to question

$\bar{z} = iz^2$

$x - iy = i(x^2 - y^2 + 2ixy)$

By comparing

$x = -2xy$ & $x^2 - y^2 = -y$

By solving $x + 2xy = 0$

We get $x = 0$ & $y = -\frac{1}{2}$

If $x = 0$ then $y = 0$ or 1

If $y = \frac{-1}{2}$ then $x = \pm\frac{\sqrt{3}}{2}$

So non-real Solution are $P(0,1), B\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$ & $R\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$

Hence area of polygon $PQR = \frac{1}{2} \times \frac{3}{2} \times \sqrt{3} = \frac{3\sqrt{3}}{4}$

12.(25) $A = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}\begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix}$

$$A^4 = \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix}\begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{4n+1} = A$$

N=1, 5, 9, ..., 97

$\Rightarrow$ total elements in the set is 25

13.(2) $z + \bar{z} = iz^2 + z^2$

Consider $z = x + iy$

$$2x = (i+1)\left(x^2 - y^2 + 2xyi\right)$$

$\Rightarrow$ $2x = x^2 - y^2 - 2xy$ *and* $x^2 - y^2 + 2xy = 0$

$\Rightarrow$ $2x = -4xy$

$\Rightarrow$ $x = 0$ *or* $y = \dfrac{-1}{2}$

Case I : $x = 0 \Rightarrow y = 0$ *here* $z = 0$

Case II : $y = \dfrac{-1}{2}$

$\Rightarrow$ $4x^2 - 4x - 1 = 0$

$(2x-1)^2 = 2$

$2x - 1 = \pm\sqrt{2}$

$x = \dfrac{1 \pm \sqrt{2}}{2}$

Here $z = \dfrac{1+\sqrt{2}}{2} - \dfrac{i}{2}$ *or* $z = \dfrac{1-\sqrt{2}}{2} - \dfrac{i}{2}$

Sum of squares of modulus of z $= 0 + \dfrac{\left(1-\sqrt{2}\right)^2 + 1}{4} + \dfrac{\left(1-\sqrt{2}\right)^2 + 1}{4} = \dfrac{8}{4} = 2$

14.(40) $1 < \left|z - 3 + 2i\right| < 4$

$\Rightarrow$ $1 < (x-3)^2 + (y+2)^2 < 16$

Shift origin at $(3,-2)$

$\Rightarrow$ $1 < x^2 + y^2 < 16$

For $x^2 + y^2 < 16 \Rightarrow (0,0)$ *and* $((0,1),(0,2),(0,3))$ on + x axis

and $(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2)$ in first quadrant.

$\Rightarrow$ $1 + 4\times 3 + 4\times 8 = 45$ *points* and for $x^2 + y^2 > 1 \Rightarrow$ remove $(0,0),(0,\pm 1)$ and $(\pm 1,0) = 5$

points. So total required points $= 45 - 5 = 40$

15.(A) $x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 + x + 1)$

$\Rightarrow$ Roots are : $\pm\omega, \pm\omega^2$

$\Rightarrow$ $\alpha^6 = 1$ and $\alpha^3 = \pm 1$

$\Rightarrow$ $\alpha^{2022} = 1$

and $\alpha^{1011} = \alpha^{3033} = \pm 1$

$\Rightarrow$ $\alpha^{1011} - \alpha^{3033} = 0$

16.(C) $|z| = 3$ represents a circle with

$C_1 : (0, 0)$

$R_1 = 3$

$\arg(3-1) - \arg(3+1) = \dfrac{\pi}{4}$

Represents the major arc of the circle (lying above x-axis) with centre.

$C_2 : (0, 1)$

$R_2 : \sqrt{2}$

$C_1C_2 = 1 < R_1 - R_2$

$\Rightarrow$ 2nd circle is fully contained inside first.

17.(A) $x^2 = 1 - 2i$

$\alpha^2 = \beta^2 = 1 - 2i$

$\alpha^8 = \beta^8 = (1-2i)^4 = 1 - 4(2i) + 6(2i)^2 - 4(2i)^3 + (2i)^4$

$= 1 - 8i - 24 + 32i + 16$

$= -7 + 24i$

$\left|\alpha^8 + \beta^8\right| = \left|-14 + 48i\right| = \sqrt{196 + 48^2} = 2\sqrt{49 + 24^2} = 50$

18.(26) $|z - 2| \le 1$ represents interior of the circle ((2, 0), 1)

Let $z = x + iy$ then

$z(1+i) + \bar{z}(1-i) \le 2$

$\Rightarrow$ $(x+iy)(1+i) + (x-iy)(1-i) \le 2$

$\Rightarrow$ $x + iy + xi - y + x - y - xi - yi \le 2$

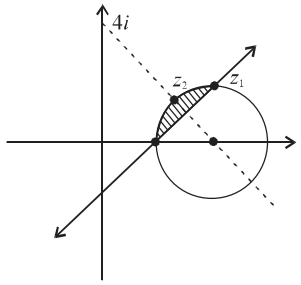
$\Rightarrow$ $x - y \le 1$

$|z - 4i|$ is maximum at $z_1(1 + 0 \cdot i)$ and minimum at $z_2\left(2 - \dfrac{1}{\sqrt{5}} + \dfrac{2i}{\sqrt{5}}\right)$

$\therefore$ $5\left(|z_1|^2 + |z_2|^2\right)$

$= 5\left(1 + \left(2 - \dfrac{1}{\sqrt{5}}\right)^2 + \dfrac{4}{5}\right)$

$= 5\left(1 + 4 + \dfrac{1}{5} - \dfrac{4}{\sqrt{5}} + \dfrac{4}{5}\right)$



$= 5\left(6 - \frac{4}{\sqrt{5}}\right) = 30 - 4\sqrt{5}$

$= \alpha + \beta\sqrt{5}$

$\alpha + \beta = 30 - 4 = 26$

19.(A) Let $z = x + iy$

$v = |z|^2 + |z-3|^2 + |z-6i|^2$

$v = x^2 + y^2 + (x-3)^2 + y^2 + x^2 + (y-6)^2$

$v = 3x^2 + 3y^2 - 6x - 12y + 45$

$v = 3(x^2 - 2x + 1) + 3(y^2 - 4y + 4) + 30$

$v = 3(x-1)^2 + 3(y-2)^2 + 30$

$v_0 = v_{\min} = 30$ when $x = 1$ and $y = 2$

$z_0 = 1 + 2i$

$\left|2z_0^2 - \bar{z}_0^3 + 3\right|^2 + v_0^2$

$\left|2(1+2i)^2 - (1-2i)^3 + 3\right|^2 + 30^2$

$= \left|8 + 6i\right|^2 + 30^2 = 100 + 900 = 1000$

20.(0) $z^2 + \bar{z} = 0$

Let $z = x + iy$

$\therefore \quad z^2 + \bar{z} = (x+iy)^2 + (x - iy)$

$= (x^2 - y^2 + x) + iy(2x - 1) = 0 + 0i$

$\therefore \quad x^2 - y^2 + x = 0$ and $y(2x-1) = 0 \quad \Rightarrow \quad y = 0,\ x = \frac{1}{2}$

If $y = 0$, then $x^2 - y^2 + x = 0$

$\Rightarrow \quad x^2 + x = 0 \quad \Rightarrow \quad x = 0, -1$

If $x = \frac{1}{2}$, $x^2 - y^2 + x = 0 \Rightarrow \quad \frac{1}{4} + \frac{1}{2} = y^2$

$y \pm \frac{\sqrt{3}}{2}$

$\therefore \quad z = 0, -1, \frac{1}{2} + \frac{\sqrt{3}i}{2}, \left(\frac{1}{2} - \frac{\sqrt{3}i}{2}\right)$

$\therefore \quad \sum_{z \in S} (\text{Re}(z) + IM(z)) = 0 - 1 + \frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{1}{2} - \frac{\sqrt{3}}{2} = 0$

21.(D) n has infinite solution.

22.(17) $A=\begin{bmatrix}2 & -1 & -1\\ 1 & 0 & -1\\ 1 & -1 & 0\end{bmatrix} \Rightarrow A^2 = A \Rightarrow A^n = A \ \forall n \in \{1,2,....100\}$

Now, $B = A - I = \begin{bmatrix}1 & -1 & -1\\ 1 & -1 & -1\\ 1 & -1 & -1\end{bmatrix}$

$B^2 = -B \quad \Rightarrow B^3 = -B^2 = B \quad \Rightarrow B^5 = B \quad \Rightarrow B^{99} = B$

Also, $\omega^{3n} = 1$

So, n = common of {1,3,5,...99} and {3,6,9,...,99} = 17

23.(6) $|z| = |z+1| \Rightarrow z = \frac{-1}{2} \pm \frac{\sqrt{3}}{2} i = \omega \text{ or } \omega^2$

$\omega^n = (-\omega^2)^n$

24.(D) Given $\left|z - \frac{1}{z}\right| = 2$

As we know

$\left|\left|z_1 - z_2\right|\right| \le \left|z_1 - z_2\right|$

$\left|\left|z\right| - \frac{1}{|z|}\right| \le 2$

$|z| = r$

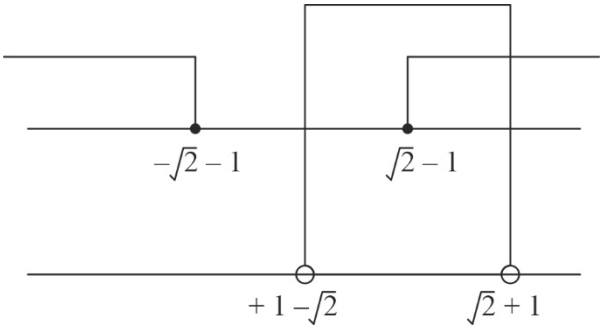
$\left|r - \frac{1}{r}\right| \le 2$

$-2 \le r - \frac{1}{r} \le 2$

$\Rightarrow \quad r - \frac{1}{r} \ge -2 \quad \& \quad r - \frac{1}{r} \le 2$

$r^2 - 1 \ge -2r \quad \& \quad r^2 - 1 - 2r \le 0$

$r^2 + 2r - 1 \ge 0 \quad \& \quad r^2 - 2r - 1 \le 0$



$r^2 + 2r - 1$

$r = \frac{-2 + \sqrt{4+4}}{2}$

$r = \frac{-2 \pm \sqrt{2}}{2}$

$r = -1 \pm \sqrt{2}$

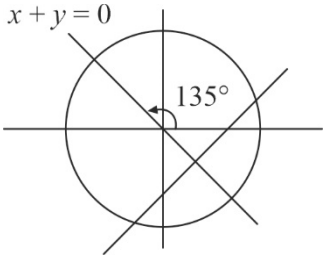
$r^2 - 2r - 1 = 0$

$r = \frac{2 \pm \sqrt{4+4}}{2}$

$r = 1 \pm \sqrt{2}$

$|z|max = 1 + \sqrt{2}$

25.(B) $|z - 1 + i| \geq |z| \Rightarrow x - y \leq 1$



$W = 2x + iy$

$2x \leq \frac{1}{2} \Rightarrow x \leq \frac{1}{4}$

$(2x)^2 + (2x)^2 < 4$

$x \in \left(\frac{-1}{\sqrt{2}}, \frac{1}{4}\right]$

26.(C) $z = x + iy$

$|z| = 2 \Rightarrow x^2 + y^2 = 4 \quad \ldots(1)$

and $|z - i| = |z + 5i|$

$\Rightarrow |x + i(y-1)| = |x + i(y+5)|$

$\Rightarrow \sqrt{x^2 + (y-1)^2} = \sqrt{x^2 + (y+5)^2}$

$\Rightarrow y^2 - 2y + 1 = y^2 + 10y + 25$

$\Rightarrow -12y = 24 \Rightarrow y = -2 \ \& \ x = 0$

So $(0, -2)$ lies on $x + 2y + 4 = 0$

27.(C) $z_1 = \frac{1 - i\sin\alpha}{1 + 2i\sin\alpha} \times \frac{1 - 2i\sin\alpha}{1 - 2i\sin\alpha}$

Given $\text{Re}(z_1) = 0$

$\Rightarrow 1 - 2\sin^2\alpha = 0 \qquad \Rightarrow \sin^2\alpha = \frac{1}{2} \qquad \Rightarrow \alpha = \frac{5\pi}{4}, \frac{7\pi}{4},\ 2\alpha = \frac{5\pi}{2}, \frac{7\pi}{2}$

$z_2 = \frac{1 + i\cos\beta}{1 - 2i\cos\beta} \times \frac{1 + 2i\cos\beta}{1 + 2i\cos\beta}$

Now, $\text{Im}\, g(z_2) = 0$

$\Rightarrow \cos\beta = 0 \; and \; \beta = \dfrac{3\pi}{2}, 2\beta = 3\pi$

$$\sum_{\alpha,\beta \in S} \left(iz_{\alpha\beta} + \frac{1}{i\bar{z}_{\alpha\beta}} \right) = \sum \left(iz_{\alpha\beta} - \frac{iz_{\alpha\beta}}{\left| z_{\alpha\beta} \right|^2} \right)$$

$$= \sum iz_{\alpha\beta} \left(\frac{1}{2} \right) = \frac{1}{2} \left[i\left(-2i \right) \right] = 1$$

28.(D) $AB = AO.z^{-i\pi/2} = -2 + i$

So $OB = \left(-2 + i \right) + \left(1 + 2i \right)$

$z_2 = -1 + 3i \qquad \therefore \quad \left| 2z_1 - z_2 \right| = \sqrt{10}$

Solutions of Archive - JEE Main & Advanced

Permutation and Combination	Class - XI | Mathematics

JEE Main 2021

1.(A) Atleast 2 Indians & twice foreigners $\therefore\ (2I, 4F)+(3I, 6F)+(4I, 8F)$

$\therefore\ {}^6C_2\ {}^8C_4 + {}^6C_3\ {}^8C_6 + {}^6C_4\ {}^8C_8 = 15\times 70 + 20\times 28 + 15\times 1 = 1625$

2.(31650)

C gets $1\ {}^{10}C_1\times(2^9-2)$

C gets $2\ {}^{10}C_2\times(2^8-2)$

C gets $3\ {}^{10}C_3\times(2^7-2)$

Add to set 31650

3.(A) $xyz = 24 = 2^3.3$

Let $x = 2^{a_1}\times 3^{b_1}$, $y = 2^{a_2}\times 3^{b_2}$ and $z = 2^{a_3}\times 3^{b_3}$

$\Rightarrow xyz = 2^{a_1+a_2+a_3}.3^{b_1+b_2+b_3} \Rightarrow a_1+a_2+a_3 = 3$ *and* $b_1+b_2+b_3 = 1 \Rightarrow\ {}^5C_2\times{}^3C_2 = 30$

4.(32) Number divisible by 3 = 18 cases

= 1,2,3,– 6 cases $= 2,3,4-6$ cases $= 1,3,5-6$ cases $= 3,4,5-6$ cases

Numbers divisible by 5 = 4 × 3 × 1 = 12

Nos divisible by both = 4

Numbers divisible by 30 or 5 = 24 + 12 – 4 = 32

5.(A) $n(A) = 3$; $x = {}^5P_3 = 60$

$n(B) = 5$; $y = {}^{15}P_3 = \dfrac{15!}{12!} = 2730 \Rightarrow 2y = 5460 = 91x.$

6.(45) For $3^n + 4^n$ to be divisible by 10.

n can be any odd number $\therefore$ Number of odd two digit numbers = 45

7.(C)

2	2	2	1	1	1	1	$\dfrac{4!}{4!\,3!} = 35$
3	2	1	1	1	1	1	$\dfrac{7!}{5!} = 42$

Total 77 ways

8.(D) $n = 2^x.3^y.5^z;\ \ y+z=5;\ \ y^{-1}+z^{-1}=\frac{5}{6}, y>z$

So; Number of odd divisors $=(1)\times(y+1)\times(z+1)=(y+1)(z+1)=1+(y+z)+(yz)$

$=1+5+6=12$ $\{\because\ y+z=5\ \ \&\ \frac{1}{y}+\frac{1}{z}=\frac{5}{6}\ \Rightarrow\frac{y+z}{yz}=\frac{5}{6}$

$\Rightarrow\ \frac{5}{yz}=\frac{5}{6}\Rightarrow yz=6\}$

9.(A) $A=\{1,2,3,.....10\}$

$f:A\to A \qquad f(k)=\begin{cases}k+1 & k \text{ is odd}\\ k & k \text{ is even}\end{cases}$

$g:A\to A \qquad g(f(x))=f(x)$

$f(1)=f(2)=2, \qquad f(3)=f(4)=4$

$f(5)=f(6)=6, \qquad f(7)=f(8)=8$

$f(9)=f(10)=10.$

$\Rightarrow \qquad g(2)=2 \qquad g(4)=4$

$g(6)=6, \qquad g(8)=8, \qquad g(10)=10.$

So remaining 5 can map to any $10.$

So number of functions g are $10\times10\times10\times10\times10=10^5$

10.(1000)

$18=2\times3^2$ $9k+3$

4 digit numbers are 9000

From	4digit number
$9k$	1000
$9k+1$	1000
'	'
'	'
'	'
$9k+8$	1000

GcD of given number & 18 is $3.$

Provided

(1) number should be odd

(2) number should be divisible by $3.$

(3) number should not be divisible by $9.$

So number can be of the form $9k+3$ or $9k+6$

So total nos. will be 2000 out of which 1000 will be even so rejected

Final answer $=1000$

11.(C) Sum of numbers $=3!(1+10+10^2+10^3).2+3(1+10+10^2+10^3)(1+3)$

$= 12\,(1111) + 12(1111) = 26664$

12.(300) Number of times 3 will be listed

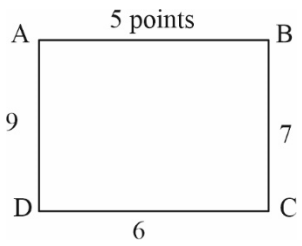
$= {}^{3}C_1 9.9 + {}^{3}C_2 \cdot 2 \cdot 9 + {}^{3}C_3 \cdot 3 \cdot 1 \;\; = 300$

13.(C) $\beta = {}^{5}C_1 \times {}^{7}C_1 \times {}^{6}C_1 \times {}^{9}C_1 = 1890$

$\alpha = {}^{5}C_1 \times {}^{7}C_1 \times {}^{6}C_1 + {}^{5}C_1 \times {}^{7}C_1 \times {}^{9}C_1 + {}^{5}C_1 \times {}^{6}C_1 \times {}^{9}C_1 + {}^{7}C_1 \times {}^{6}C_1 \times {}^{9}C_1$

$= 210 + 315 + 270 + 378$

$\beta - \alpha = 717$



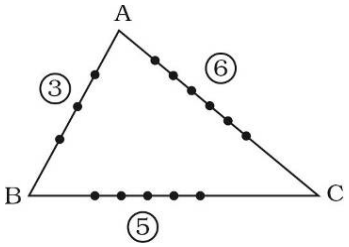
14.(C) $7 \times 4 + 6n = 52$

$6n = 24$

$n = 4$

15.(B) Total number of points $= 3 + 5 + 6 = 14$

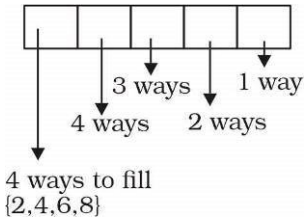
Required result $= {}^{14}C_3 - {}^{3}C_3 - {}^{5}C_3 - {}^{6}C_3 = 333$



16.(777) Total number of ways for at least 4 bowlers, 5 batsman and 1 wicketkeeper

$= {}^{6}C_4 \left({}^{7}C_6 \times {}^{2}C_1 + {}^{7}C_5 \times {}^{2}C_2 \right) + {}^{6}C_5 \times {}^{7}C_5 \times {}^{2}C_1 = 777$

17.(96) $0, 2, 4, 6, 8$



$\Rightarrow$ Required Ans $= 4 \times 4 \times 3 \times 2 \times 1 = 96$

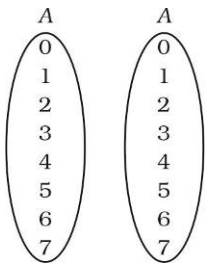
18.(720)

$f(1) + f(2) + f(3) = 3$ ....(i)

Since, function is bijective i.e.

one – one and onto;

$1, 2, 3 \in$ Domain, must have their individual different images in $0, 1, 2 \in$



Co-domain

{$\because$ (i) must be true}

This can be possible by $3!$ ways.

Now, rest $0, 4, 5, 6, 7 \in$ domain, must be connected with $3, 4, 5, 6, 7$ one to one by $5!$ ways

$\therefore$ Required Ans $= 3!5! = 6 \times 120 = 720$

19.(238) Number of students in class $10^{th} = 5$

Number of students in class $11^{th} = 6$

Number of students in class $12^{th} = 8$

Possible selection according to the question

(a) 2 from 10th
3 from 11th
5 from 12th
Number of ways in this case
$^5C_2 \times {}^6C_3 \times {}^8C_5$

(b) 2 from 10th
2 from 11th
6 from 12th
Number of ways in their case
$^5C_2 \times {}^6C_2 \times {}^8C_6$

(c) 3 from 10th
2 from 11th
5 from 12th

Number of ways in this case
$^5C_3 \times {}^6C_2 \times {}^8C_5$

So total number of ways $= {}^5C_2 \times {}^6C_3 \times {}^8C_5 + {}^5C_2 \times {}^6C_2 \times {}^8C_6 + {}^5C_3 \times {}^6C_2 \times {}^8C_5 = 23800$

So required answer is 238.

20.(C) ${}^nC_r = {}^nC_{r-1} \Rightarrow r + r - 1 = n \Rightarrow 2r = n + 1$ and ${}^nP_r = {}^nP_{r+1}$

$\Rightarrow n - r = 1$ or $n = r + 1 \Rightarrow 2r = r + 1 + 1 \Rightarrow r = 2$ and $n = 3$

21.(490) $F(mn) = f(m) \cdot f(n)$

Put $m = 1$ $f(n) = f(1) \cdot f(n) \Rightarrow f(1) = 1$

Put $m = n = 2$

$$f(4) = f(2) \cdot f(2) \begin{cases} f(2) = 1 \Rightarrow f(4) = 1 \\ \quad or \\ f(2) = 2 \Rightarrow f(4) = 4 \end{cases}$$

Put $m = 2, n = 3$

$$f(6) = f(2) \cdot f(3) \begin{cases} when\ f(2) = 1 \\ f(3) = 1\ to\ 7 \\ \\ f(2) = 2 \\ f(3) = 1\ or\ 2\ or\ 3 \end{cases}$$

$f(5),\ f(7)$ can take any value

Total $= (1 \times 1 \times 7 \times 1 \times 7 \times 1 \times 7) + (1 \times 1 \times 3 \times 1 \times 7 \times 1 \times 7) = 490$

22.(924) $2^{10}\ 5^{10}\ (4k_2 - 1)^{11}\ (4k_3 + 1)^{13}$

$2^{10}\ (4k_1 + 1)^{10}\ (4k_2 - 1)^{11}\ (4k_3 + 1)^{13}$

Number of divisor $= (11 \times 6 \times 14) = 924$

23.(100) $\boxed{5}\boxed{a}\boxed{b}\boxed{b}\boxed{a}\boxed{5}$

It is always divisible by 5 and 11.

$\therefore$ Required number $= 10 \times 10 = 100$

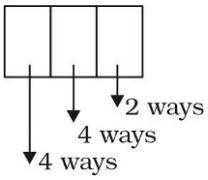
24.(52) 0, 1, 3, 4, 5, 6, 7

We need to find three digit even numbers

Case I: 4 or 6 are in unit's place

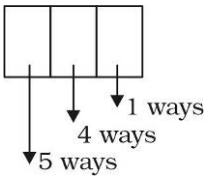
i.e. 2 ways to fill unit's place

& 4 ways to fill hundred's place because zero can not be placed here & 4 ways to fill 10's place



$\Rightarrow$ Total no. $s = 4 \times 4 \times 2 = 32$

Case II: Zero is fixed at unit's place.



Hence, required number of numbers = 32 + 20 = 52

25.(77) Words starting with $A \equiv \underline{A}$ _____ ____ __

$$\equiv \frac{5!}{2!} = 60$$

R R together $\equiv 4!$

$= 60 - 4! = 36$

Words starting with $E = 36$

Words starting with FAE and RR not together $\equiv \underline{F}\ \underline{A}\ \underline{E}\ \_\ \_\ \_ \equiv 1$

Words starting with FAM and RR not together $\equiv \underline{F}\ \underline{A}\ \underline{M}\ \_\ \_\ \_ \equiv 1$

Words starting with FARE RR not together $\underline{F}\ \underline{A}\ \underline{R}\ \underline{E}\ \_\ \_\ \_ \equiv 2$

Rank of FARMER $= 36 + 36 + 1 + 1 + 2 + 1 = 77$

26.(D) No. of distinct triangles $= {}^{15}C_3$

No. of triangles for which $i + j + k = 15$ is 12

Ans. ${}^{15}C_3 - 12 = 443$

27.(576)

All consonants should not be together = Total – All consonants together = 6! – 3! 4! = 576

JEE Advanced 2021

1.(ABD) $n_1 = 10^3 = 1000$

For n_2: Let $j = j + 2 \quad \Rightarrow \quad j \in \{3,.....10\}$

Because $j \geq 1$ and $j + 2 \leq 10$

For $i < j$', pick any 2 no. s from {1, 2... 10} except (1, 2) $\Rightarrow \quad n_2 = {}^{10}C_2 - 1 = 44$

$n_3 = {}^{10}C_4$

$n_4 = {}^{10}C_4 \times 4!$

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Solutions of Archive - JEE Main & Advanced

Permutation and Combination	Class - XI | Mathematics

JEE Main 2022

1.(576) Let the number be *abcdefg*

$\therefore (a+c+e+g)-(b+d+f)=11K$

$a+b+c+d+e+f+g=31$

$\therefore a+c+e+g=21$ & $b+d+f=10$

or $a+c+e+g=10$ & $b+d+f=21$

If $a+c+e+g=21$ & $b+d+f=10$

$(b,d,f)\in\{(1,2,7),(2,3,5),(1,4,5)\}$

$(a,c,e,g)\in\{(1,4,7,9),(3,4,5,9),(2,3,7,9)\}$

$\therefore$ Total $=(3!\times 3)(4!)=432$

Now if $a+c+e+g=10$ & $b+d+f=21$

Then, $(a,c,e,g)\in\{(1,2,3,4)\}$

$(b,d,f)\in\{(5,7,9)\}$

$\therefore$ Total $=3!\times 4!=144$

$\therefore$ Total numbers $=432+144=576$

2.(1633) Sum $=(1+2+\ldots+100)-(2+4+\ldots+100)-(3+6+\ldots+99)+(6+12+\ldots+96)$

$$=\frac{100\times 101}{2}-\frac{2\times 50\times 51}{2}-\frac{3\times 33\times 34}{2}+6\left(\frac{16\times 17}{2}\right)$$

$=5050-2550-1683+816=1633$

3.(40) $x_1+x_2+x_3+x_4+x_5=5$

There is only one possibility i.e.

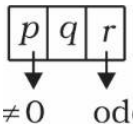
$3,3,3,-2,-2$

$\therefore$ Number of ways is $=\frac{5!}{3!2!}2\times 2=40$

4.(243) $1--\rightarrow 27$ 3 digit no, starting with 1 will be 27.

$27\times 9=243$

5.(63)



Maximum sum of digits = 27

Possible sum of digits which are divisible by $7 = 7,14,21$

C-I sum $= 7$:

$r = 1 \Rightarrow p + q = 6 \Rightarrow (p,q) \equiv (1,5),(2,4),(3,3),(4,2),(5,1),(6,0)$

$r = 3 \Rightarrow p + q = 4 \Rightarrow (p,q) \equiv (1,3),(2,2),(3,1),(4,0)$

$r = 5 \Rightarrow p + q = 2 \Rightarrow (1,1),(2,0)$

C-II: sum $= 14$:

$r = 1 \Rightarrow p + q = 13 \Rightarrow (4,9)(5,8)(6,7),(7,6)(8,5)(9,4)$

$r = 3 \Rightarrow p + q = 11 \Rightarrow (5,6)(6,5)(7,4),(8,3)(9,2)(3,8)(4,7)(2,9)$

$r = 5 \Rightarrow p + q = 9 \Rightarrow (1,8),(2,7),(3,6),(4,5),(5,4),(6,3)(7,2),(8,1),(9,0)$

$r = 7 \Rightarrow p + q = 7 \Rightarrow (1,6),(2,5),(3,4),(4,3),(5,2),(6,1),(7,0)$

$r = 9 \Rightarrow p + q = 5 \Rightarrow (1,4),(2,3),(3,2),(4,1),(5,0)$

C-III; sum $= 21$:

$r = 3 \Rightarrow p + q = 18 \Rightarrow (9,9)$

$r = 5 \Rightarrow p + q = 16 \Rightarrow (7,9),(8,8),(9,7)$

$r = 7 \Rightarrow p + q = 14 \Rightarrow (5,9),(6,8),(7,7),(8,6),(9,5)$

$r = 9 \Rightarrow p + q = 12 \Rightarrow (3,9),(4,8),(5,7),(6,6),(7,5),(8,4),(9,3)$

Hence, total = 63

6.(150)

$\gcd(n, 36) = 2$

$\Rightarrow$ $2n$ *and* $4n$ *and* $3n$

And n is *a* 3 digits

$n = 106, 110, 118, 122, 130, 134, 142 = 150$

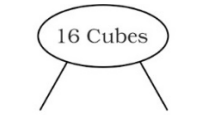
7.(1120)

Total number of ways

$= {}^{10}C_3 \cdot {}^5C_3 - {}^8C_1 \cdot {}^5C_3$

$= 1120$

8.(56)



$R\ BB\ R\ BB\ R\ BB\ R\ BB\ R$

$\downarrow x_1 \quad \downarrow x_2 \quad \downarrow x_3 \quad \downarrow x_4 \quad \downarrow x_5 \quad \downarrow x_6$

No. of ways is equivalent to no. of non-negative solution of equation.

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 3$

$n = 6; r = 3$

${}^{n+r-1}C_r = {}^{6+3-1}C_3 = 56$

9.(D) $x_1 + x_2 + x_3 + x_4 = 30$

$4 \le x_2 \le 7,\ 2 \le x_3 \le 6, x_1, x_4 \ge 0.$

Coffs of $x^{30} in \left(x^0 + x^1 + x + ... + x^{30}\right)^2 \left(x^4 + x^5 + x^6 + x^7\right)\left(x^2 + x^3 + x^4 + x^5 + x^6\right)$

Coffs of $x^{24} in \left(\frac{1-x^{31}}{1-x}\right)^2 \left(1 + x + x^2 + x^3\right)\left(1 + x + x^2 + x^3 + x^4\right)$

Coffs of $x^{24} in \left(1-x\right)^{-2}\left(\frac{1-x^4}{1-x}\right)\left(\frac{1-x^5}{1-x}\right)$

Coffs of $x^{24}\ in \left(1-x\right)^{-4}\left(1 - x^4 - x^5 + x^9\right).$

${}^{4+24-1}C_{24} - {}^{4+20-1}C_{20} - {}^{4+19-1}C_{19} + {}^{4+5-1}C_{15-1}.$

${}^{27}C_{24} - {}^{23}C_{20} - {}^{22}C_{19} + {}^{18}C_{14} = 430.$

10.(0) $20 = \frac{\sum_{i=1}^{7} |x_i - 62|}{7}$

$\Rightarrow \left|x_1 - 62\right|^2 + \left|x_2 - 62\right|^2 + ... + \left|x_7 - 62\right|^2 = 140$

Now if any student have less than 50 marks RHS becomes negative, hence no student can get less than 50 marks $\therefore$ No student will fail.

11.(B) For non existance of solution

$$\Delta = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & |\lambda| \end{vmatrix} = 0 \Rightarrow \lambda = \pm 7$$

$$\Delta_1 = \begin{vmatrix} -2 & 3 & -1 \\ 4 & 1 & 1 \\ 4\lambda - 4 & -1 & |\lambda| \end{vmatrix}$$

For $\lambda = 7$

$$\Delta_1 = \begin{vmatrix} -2 & 3 & -1 \\ 4 & 1 & 1 \\ 24 & -1 & 7 \end{vmatrix} = 0$$

For $\lambda = -7$

$$\Delta_1 = \begin{vmatrix} -2 & 3 & -1 \\ 4 & 1 & 1 \\ -32 & -1 & 7 \end{vmatrix} \neq 0$$

Thus for $\lambda = -7$, the given set of linear equation has no solution.

12.(D) Set -1 $\{1,2,5,6,7\}$

Set-2 $\{1,2,3,5,7\}$

We have to form even number with set 1 and set -2 without replacement taking all at a time.

When we use set -1

Total number of number $= 4! \times 2$

When we use set -2

Total number of number $= 4! \times 1$

Then total number of number $= 48 + 24 = 72$

13.(1086)

Possible First three digits	**Last digit**	**No. of 4-digit numbers**
0, 1, 2, 3, ..., 9	1	9 × 10 × 10
0, 2, 4, 6, 8	2	4 × 5 × 5
0, 3, 6, 9	3	3 × 4 × 4
0, 4, 8	4	2 × 3 × 3
0, 5	5	1 × 2 × 2
0, 6	6	1 × 2 × 2
0, 7	7	1 × 2 × 2
0, 8	8	1 × 2 × 2
0, 9	9	1 × 2 × 2
Total no. of 4-digit numbers		= 1086

14.(37830)

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A) =$ Number of ways in which b_1, b_2 and b_3 are consecutive.

3 consecutive numbers can be selected in 98 ways.

Now these 3 consecutive numbers can be arranged in 2 ways in ascending or descending.

Now b_4 can take its value in 97 ways.

$\therefore \quad n(A) = 98 \times 2 \times 97$

$n(B) =$ Number of ways in which b_2, b_3 and b_4 are consecutive.

3 consecutive numbers can be selected in 98 ways.

Now these 3 consecutive numbers can be arranged in 2 ways in ascending or descending.

Now b_1 can take its value in 97 ways.

$\therefore \quad n(B) = 98 \times 2 \times 97$

$n(A \cap B) =$ Number of ways in which b_1, b_2, b_3 and b_4 are consecutive.

4 consecutive numbers can be selected in 97 ways.

Now these 4 consecutive numbers can be arranged in 2 ways in ascending or descending.

$\therefore \quad n(A \cap B) = 97 \times 2$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= (98 \times 2 \times 97) + (98 \times 2 \times 97) - (97 \times 2) = 37830$

15.(7073)

Total number of required passwords

= Total number of passwords – {Total number of passwords which are made up using $\{1,2,3,4,5\}$ only}

$= \left(10^6 - 5^6\right) + \left(10^7 - 5^7\right) + \left(10^8 - 5^8\right)$

$= 10^6\left(1+10+100\right) - 5^6\left(1+5+25\right) = 7073 \times 5^6 \Rightarrow \alpha = 7073$

16.(1492)

M	A	N	K	I	N	D

$\left(\frac{4\times 6!}{2!}\right) + (5!\times 0) + \left(\frac{4!\times 3}{2!}\right) + (3!\times 2) + (2!\times 1) + (1!\times 1) + (0!\times 0) + 1 = 1492$

17.(6) For divisibility by 55

Number should be divisible by 5 & 11

$\boxed{a}\boxed{b}\boxed{c}\boxed{5}$

Now, $a + c = b + 5$

For $\boxed{b=1} \Rightarrow \boxed{\begin{matrix} a=2, & c=4 \\ a=4, & c=2 \end{matrix}}$

For $\boxed{b=2} \Rightarrow \boxed{a=3, c=4}$ or $\boxed{a=4, c=3}$

For $\boxed{b=3} \Rightarrow \boxed{\begin{matrix} a=6, & c=2 \\ a=2, & c=6 \end{matrix}}$

'5' digit number is not possible

Hence '6' possible four digit

Number are divisible by '55' $\boxed{6}$

18.(30) Number formed by last 2 digits should be divisible by 4.

List 2 digit	1, 2, 3, 4, 5, 6	1, 2
_ _ 12	0	
$\boxed{2}$ _ 16	1×3	$= 3$
$\boxed{1}$ _ 24	1×3	$= 3$
$\boxed{1}$ _ 32	1×3	$= 3$
36	2×3	$= 6$
$\boxed{1}$ _ 52	1×3	$= 3$
56	2×3	$= 6$
64	2×3	$= 6$
		Total 30

19.(180) $3 \times \frac{5!}{2!2!} + \frac{5!}{3!\times 2!} + \frac{5!}{2!} + \frac{5!}{3!} = 180$

Solutions of Archive - JEE Main & Advanced

Binomial Theorem	Class - XI | Mathematics

JEE Main 2021

1.(A) $\left({}^{-15}C_1+2\cdot{}^{15}C_2-3\cdot{}^{15}C_3+.....15\cdot{}^{15}C_{15}\right)+\left({}^{14}C_1+{}^{14}C_3+{}^{14}C_5+.....+{}^{14}C_{11}\right)$

$=\sum_{r=1}^{15}(-1)^r\cdot r\cdot{}^{15}C_r+\left({}^{14}C_1+{}^{14}C_3+{}^{14}C_5+.....+{}^{14}C_{11}+{}^{14}C_{13}\right)-{}^{14}C_{13}$

$=15\sum_{r=1}^{15}(-1)^r\cdot{}^{14}C_{r-1}+(2^{13}-14)=15\left[-{}^{14}C_0+{}^{14}C_1+.....+{}^{14}C_{14}\right]+2^{13}-14=15(0)+2^{13}-14$

$=2^{13}-14$

2.(B) ${}^2C_2={}^3C_3$

${}^3C_3+{}^3C_2={}^4C_3$

${}^4C_3+{}^4C_2={}^5C_3$

.

.

.

${}^nC_3+{}^nC_2={}^{n+1}C_3$

${}^{n+1}C_2+2\,{}^{n+1}C_3={}^{n+1}C_2+{}^{n+1}C_3+{}^{n+1}C_3={}^{n+2}C_3+{}^{n+1}C_3$

$=\frac{\lfloor n+2}{6\lfloor n-1}+\frac{\lfloor n+1}{6\lfloor n-2}=\frac{1}{6}\left((n+2)(n+1)(n)+(n+1)(n)(n-1)\right)=\frac{1}{6}n(n+1)\left[n+2+n-1\right]$

$=\frac{1}{6}\cdot n\cdot(n+1)(2n+1)$

3.(12) The given expression is:

$$\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i}+\sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i}$$

Since $\binom{n}{r}=0$ for $n < r$. so no maximal value of k exists.

For example, let $k = 1000$, $\binom{10}{i}=0$ for all $i > 10$

So, no upper bound on k exists.

Note: They state that the largest value of k is 12. But for k = 12 some the binomial coefficients of $\sum_{i\ 0}^{k}\binom{10}{i}\binom{15}{k-i}$ must be zero. The same argument can be extended to k = 1000 and so on.

4.(1) Let $x = 4k+3$

$\therefore \quad (2020+x)^{2022} = (2020+4k+3)^{2022}$

$= (4(505)+4k+3)^{2022} = (4P+3)^{2022} = (4P+4-1)^{2022} = (4A-1)^{2022}$

$= 2022C_0(4A)^{2022} - {}^{2022}C_1(4A)^{2021} + ... - {}^{2022}C_{2021}(4A) + {}^{2022}C_{2022} \quad = 8\lambda+1$

$\therefore$ Remainder = 1

5.(D) General term ${}^{10}C_r\left(tx^{1/5}\right)^{10-r}\cdot\left(\frac{(1-x)^{1/10}}{t}\right)^r$

Effective power of $t = 10-r-(r) = 10-2r$

To be independent of $t \to 10-2r = 0$

$r = 5$

${}^{10}C_5 x(1-x)^{1/2}$ is the term independent of t

$y = x(1-x)^{1/2}$ to be maxima at $x = \frac{2}{3}$

So max value $= {}^{10}C_5\frac{2}{3}\left(\frac{1}{3}\right)^{1/2} = \frac{\lfloor 10}{(\lfloor 5)^2}\cdot\frac{2}{3\sqrt{3}}$

6.(45) $A = 30\cdot{}^{30}C_0 + 29\cdot{}^{30}C_1 + ... + 2\cdot{}^{30}C_{28} + 1\cdot{}^{30}C_{29}$

$\because \quad {}^nC_r = {}^nC_{n-r}$

$A = 30\cdot{}^{30}C_{30} + 29\cdot{}^{30}C_{29} + ... + 2\cdot{}^{30}C_2 + 1\cdot{}^{30}C_1$

$= 30\times\frac{30}{30}\cdot{}^{29}C_{29} + 29\times\frac{30}{29}\times{}^{29}C_{28} + 2\times\frac{30}{2}\times{}^{29}C_1 + 1\times\frac{30}{1}\times{}^{29}C_0$

$\because \quad {}^nC_r = \frac{n}{r}{}^{n-1}C_{r-1}$

$A = 30\left[{}^{29}C_{29} + {}^{29}C_{28} + ... + {}^{29}C_1 + {}^{29}C_0\right] = 30.2^{29} = 15.2^{30} \quad \Rightarrow \quad m+n = 45$

7.(A) $\sum_{r=0}^{6}{}^6C_r\cdot{}^6C_{6-r} = {}^6C_0\,{}^6C_6 + {}^6C_1\,{}^6C_5 + ... + {}^6C_6\,{}^6C_0 = {}^{12}C_6 = \frac{12!}{6!6!} = 924$

8.(4) $\left(x+\frac{a}{x^2}\right)^n$

$T_{r+1} = {}^nC_r\,x^{n-r}\left(\frac{a}{x^2}\right)^r = {}^nC_r\,a^r\,x^{n-3r}$

$${}^nC_2 a^2 : {}^nC_3 a^3 : {}^nC_4 a^4 = 12 : 8 : 3$$

$$\frac{{}^nC_3 a^3}{{}^nC_2 a^2} = \frac{8}{12}$$

$$\frac{n-3+1}{3}a = \frac{2}{3} \qquad \textbf{...(i)}$$

$$\frac{{}^nC_4 a^4}{{}^nC_3 a^3} = \frac{3}{8}$$

$$\frac{n-4+1}{4}a = \frac{3}{8} \qquad \textbf{...(ii)}$$

(i) / (ii)

$$\frac{n-2}{3\frac{(n-3)}{4}} = \frac{2}{3}\times\frac{8}{3}$$

$$\frac{4(n-2)}{3(n-3)} = \frac{2\times 8}{3\times 3}$$

$$\frac{(n-2)}{(n-3)} = \frac{4}{3}$$

$$3n-6 = 4n-12 \quad \Rightarrow \quad n = 6$$

$$\frac{4}{3}a = \frac{2}{3} \;\Rightarrow\; a = \frac{1}{2}$$

$$n - 3r = 0$$

$$r = \frac{n}{3} = 2$$

Required term $= {}^6C_2\left(\frac{1}{2}\right)^2 = \frac{15}{4} = 3.75 \approx 4$

9.(A) $(1+x+2x^2)^{20} = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{40}x^{40}$

$$\sum_{r=0}^{40} a_r = (4)^{20} = 2^{40}$$

$$\sum_{r=0}^{40} (-1)^r a_r = 2^{20}$$

$$\Rightarrow \qquad 2\sum_{r=0}^{18} a_{2r+1} = 2^{40} - 2^{20} = 2^{20}(2^{20}-1) \qquad \Rightarrow \qquad \sum_{r=0}^{18} a_{2r+1} = 2^{19}(2^{20}-1)$$

Now a_{39} is the coefficient of x^{39} in

$$(1+x+2x^2)^{20} = (1+x+2x^2)(1+x+2x^2)\ldots\ldots(1+x+2x^2)$$

$$= {}^{20}C_1 \cdot 2^{19} = 20 \cdot 2^{19}$$

Required sum $= 2^{19}(2^{20}-1) - 20\cdot 2^{19}$

$$= 2^{19}(2^{20} - 21)$$

10.(6)

$$A=\sum_{k=0}^{n}(-1)^k\,{}^nC_k\left(\frac{1}{2}\right)^k+\sum_{k=0}^{n}(-1)^k\,{}^nC_k\left(\frac{3}{4}\right)^k+\sum_{k=0}^{n}(-1)^k\,{}^nC_k\left(\frac{7}{8}\right)^k+\sum_{k=0}^{n}(-1)^k\,{}^nC_k\left(\frac{15}{16}\right)^k+\sum_{k=0}^{n}(-1)^k\,{}^nC_k\left(\frac{31}{32}\right)^k$$

$$A=\left(1-\frac{1}{2}\right)^n+\left(1-\frac{3}{4}\right)^n+\left(1-\frac{7}{8}\right)^n+\left(1-\frac{15}{16}\right)^n+\left(1-\frac{31}{32}\right)^n$$

$$A=\frac{1}{2^n}+\frac{1}{4^n}+\frac{1}{8^n}+\frac{1}{16^n}+\frac{1}{32^n};\quad A=\frac{1}{2^n}\left(\frac{1-\left(\frac{1}{2^n}\right)^5}{1-\frac{1}{2^n}}\right)=\frac{1-\frac{1}{2^{5n}}}{2^n-1};\quad \left(2^n-1\right)A=1-\frac{1}{2^{5n}}$$

It is given that

$$63A=1-\frac{1}{2^{30}};\quad 5n=30;\qquad n=6$$

11.(4) $=(2021)^{3762}=(17\times119-2)^{3762}=(-2)^{3762}=2^2\left[2^{3760}\right]$

$4[17-1]^{140}=4$ (remainder)

12.(C) $T_4={}^7C_3\,x^4.\left(x^{10g_2^x}\right)^3=4480$

$x^4.x^{3\log_2^x}=\frac{4480}{35}=128$

$x^{4+\log_2^{x^3}}=128$

$x=2$

13.(BONUS)

Question incomplete, n should be given as 10

14.(160) $\sum_{r=1}^{10}r!(r^3+6r^2+2r+5)=\sum_{r=1}^{10}r!((r+1)(r+2)(r+3)-9r-1)$

$$=\sum_{r=1}^{10}(r+3)!-9(r+1)!+8r!=\sum_{r=1}^{10}\{(r+3)!-(r+2)!\}+\{(r+2)!-(r+1)!\}-8\{(r+1)!-r!\}$$

$$=13!-3!+12!-2!-8(11!-1!)=13!+12!-8\times11!-3!-2!+8=(13\times12+12-8)11!=(160)11!$$

$\alpha=160$

15.(210) $\left[\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right]^{10}=\left[(x^{1/3}+1)-(1+x^{-1/2})\right]^{10}=\left[x^{1/3}-x^{-1/2}\right]^{10}$

General term $={}^{10}C_r\cdot(x^{1/3})^{10-r}\cdot(-x^{-1/2})^r={}^{10}C_r\cdot x^{\frac{10-r}{3}}\cdot(-1)^r\cdot r^{-r/2}=(-1)^r\cdot{}^{10}C_r\cdot x^{\frac{10-r}{3}-\frac{r}{2}}$

$\frac{10-r}{3}-\frac{r}{2}=0\Rightarrow\frac{10-r}{3}=\frac{r}{2}\quad\Rightarrow 20-2r=3r\quad\Rightarrow r=4$

Term independent of $x=(-1)^4\cdot{}^{10}C_4\cdot x^0={}^{10}C_4=210$

16.(D) $T_{r+1} = {}^{60}C_r \left(3^{\frac{1}{4}}\right)^{60-r} \left(5^{\frac{1}{8}}\right)^{r} \quad \Rightarrow \quad T_{r+1} = {}^{60}C_r\, 3^{\frac{60-r}{4}}\, 5^{\frac{r}{8}}, 0 \le r \le 60$

For rational terms $r = 0,8,16,.....,56$

Total terms = 61

$\therefore$ Total Irrational terms = $61-8=53$ $\quad\therefore\quad$ $n-1=52$ which is divisible by 26

17.(A) $\because \quad (1-x+x^3)^n = a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_{3n}x^{3n}$

Put $x=1, \quad 1 = a_0 + a_1 + a_2 + a_3 + a_4 + ... + a_{3n}$

Put $x=-1 \quad 1 = a_0 - a_1 + a_2 - a_3 + a_4 ..... + (-1)^{3n} a_{3n}$

Solving, we get $a_0 + a_2 + a_4 + ... = 1$

And $\quad a_1 + a_3 + a_5 + ... = 0$

Now $\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4 \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} = (a_0 + a_2 + a_4 + ...) + 4(a_1 + a_3 + a_5 ....) = 1$

18.(B) Given that $(1-x)^{101}\left(x^2+x+1\right)^{100} = (1-x)\left(1-x^3\right)^{100}$

$= (1-x)\left\{{}^{100}C_0 - {}^{100}C_1x^3 + {}^{100}C_2\,x^6 + ... + {}^{100}C_{84}\,x^{252} - {}^{100}C_{85}\,x^{255} + {}^{100}C_{86}\,x^{258} ...\right\}$

$\therefore$ coefficient of $x^{256} = {}^{100}C_{85} = {}^{100}C_{15}$

19.(21) $\because \quad \left(4^{\frac{1}{4}} + 5^{\frac{1}{6}}\right)^{120} = \left(2^{\frac{1}{2}} + 5^{\frac{1}{6}}\right)^{120} \quad\therefore\quad T_{r+1} = {}^{120}C_r \left(2^{\frac{1}{2}}\right)^{120-r} (5)^{\frac{r}{6}}$

For rational terms, $r = 6\lambda, 0 \le r \le 120$ $\therefore$ Total Number of terms = 21

20.(B) $\left(1 - my + m_{c_2}y^2 ....\right)\left(1 + xy + x_{c_2}y^2\right)$

$a_1 = -m + n = 10 \;|\; a_2 = m_{c_2} + n_{c_2} - mn = 10$

$n = 10 + m \quad \frac{m(m-1)}{2} + \frac{n(n-1)}{2} - mn = 10$

$\frac{m(m-1)}{2} + \frac{(10+m)(g+m)}{2} - m(10+m) = 10$

$m^2 - m + m^2 + 19m + 50 - 20m - 2m^2 = 20$

$-2m = -70; \quad m = 35$

$n = 45; \; m+n = 80$

21.(96) $n \in \{1,2,3,....,100\}$

$11^n - 9^n > 10^n \quad \Rightarrow \quad (10+1)^n - (10-1)^n > 10^n$

$\Rightarrow \quad \left\{{}^nC_0(10)^n + {}^nC_1 10^{n-1} + ... + {}^nC_n 10^0\right\} -$

$\left\{{}^nC_0(10)^n - {}^nC_1(10)^{n-1} + {}^nC_2(10)^{n-2} + ... + (-1)^n\,{}^nC_n 10^0\right\} > 10^n$

$\Rightarrow \quad 2\left[{}^nC_1 10^{n-1} + {}^nC_3 10^{n-3} + ...\right] > 10^n$

$\Rightarrow \quad \frac{2}{10}\left[{}^{n}C_1 10^n + {}^{n}C_3 10^{n-2} + ...\right] > 10^n \Rightarrow \frac{1}{5}\left[{}^{n}C_1 + {}^{n}C_3 10^{-2} + {}^{n}C_5 10^{-4} + ...\right] > 1$

Clearly; $n \geq 5$ satisfies this.

$\Rightarrow$ Req. n are 5,6,7,8,.....,100 $\therefore$ Required no. of elements $= 96$

22.(8) $T_{\lambda+1} = {}^{10}C_\lambda \left(2x^r\right)^{10-\lambda}\left(\frac{1}{x^2}\right)^{\lambda} = {}^{10}C_\lambda 2^{10-\lambda} x^{10r-r\lambda-2\lambda}$

Put $10r - r\lambda - 2\lambda = 0 \Rightarrow r = \frac{2\lambda}{10-\lambda}$ ...(i)

Now; $T_{\lambda+1} = 180$ (when power of x is zero)

$\Rightarrow \quad {}^{10}C_\lambda 2^{10-\lambda} = 180 \Rightarrow \lambda = 8$ ...(ii)

(ii) in (i) $\Rightarrow \quad r = 8$

23.(D) *If* $x \to 0 \Rightarrow \frac{1}{1-x} = 1 + x + x^2 + x^3 .....\infty$

$$\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + ... + \frac{1}{a-nb}$$

$$= \frac{1}{a}\left[\left(1 + \frac{b}{a} + \left(\frac{b}{a}\right)^2 + \left(\frac{b}{a}\right)^3 ....\right) + \left(1 + \frac{2b}{a} + \frac{4b^2}{a^2} + \frac{(8b)^3}{a^3} + ...\right) + \left(1 + \frac{nb}{a} + \frac{(nb)^2}{a^2} + \frac{(nb)^3}{a^3} + ..\right)\right]$$

Ignore cube and high power

$$s = \frac{1}{a}\left[n + (1+2+3+....n)\frac{b}{a} + \left(1 + 2^2 + 3^2 + ...n^2\right)\frac{b^2}{a^2}\right]$$

$$s = \frac{1}{a}\left[n + \frac{n(n+1)}{2}.\frac{b}{a} + \frac{n(n+1)(2n+1)}{6}.\frac{b^2}{a^2}\right]$$

$\gamma =$ coefficient of $n^3 = \frac{1}{a}\left[\frac{2b^2}{6a^2}\right] = \frac{b^2}{3a^3}$

24.(1) Coefficient of middle term in $(1+x)^{20}$ is ${}^{20}C_{10}$

Sum of coefficient of two middle terms in $(1+x)^{19}$ is ${}^{19}C_9 + {}^{19}C_{10}$

So $\frac{{}^{20}C_{10}}{{}^{19}C_9 + {}^{19}C_{10}} = \frac{{}^{20}C_{10}}{{}^{20}C_{10}} = 1$

25.(210) $$\left(\frac{(x+1)}{\left(x^{2/3} - x^{1/3}\right)+1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$$

$$\left(\left(x^{1/3}+1\right) - \frac{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}{\sqrt{x}\left(\sqrt{x}-1\right)}\right)^{10}$$

$$\left(x^{1/3} - x^{-1/2}\right)^{10}$$

$T_{r+1} = {}^{10}C_r x^{\frac{10-r}{3}} \left(-x^{-1/2}\right)^r$

For $r = 4$ coefficient of x becomes 0.

So term in dependent of x is $T_5 = {}^{10}C_4 = 210$

26.(C) Consider $\left(1+\frac{1}{n}\right)^n = 1 + {}^nC_1 \cdot \frac{1}{n} + {}^nC_2 \cdot \frac{1}{n^2} + {}^nC_3 \cdot \frac{1}{n^3} + \ldots\ldots + {}^nC_n \cdot \frac{1}{n^n}$

Consider: $\frac{{}^nC_r}{n^r} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!\, n^r} = (1)\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\left(1-\frac{(r-1)}{n}\right)\frac{1}{r!} \le \frac{1}{r!}$

So, $1 + \frac{{}^nC_1}{n} + \frac{{}^nC_2}{n^2} + \ldots\ldots + \frac{{}^nC_n}{n^n}$

$\le 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots\ldots + \frac{1}{n!} < e$

and obviously the number is greater than 2 $\Rightarrow$ $\left[\left(n+\frac{1}{n}\right)^n\right] = 3$

27.(D) For $\sin(2\alpha) = 1$, the maximum occurs at $a = 2$ and for $\sin(2\alpha) = -1$, the max. value of the function Occurs at $a = -2$.

The maximum value of the function is $\frac{10!}{(5!)^2}\left|\frac{a}{2}\right|^5$ NOT $\frac{10!}{(5!)^2}\left(\frac{a}{2}\right)^5$

28.(D) $T_{r+1} = {}^{12}C_r (2)^{\frac{12-r}{3}} (3)^{\frac{r}{4}}$

r is a multiple of 3 as well as 4 $\Rightarrow$ r must be a multiple of 12 $\Rightarrow$ r = 0 and r = 12

Sum = ${}^{12}C_0(2)^4 + {}^{12}C_{12}(3)^3 = 16 + 27 = 43$

29.(98) $\sum_{r=0}^{n}(2r+1)\ {}^nC_r = 2^{100}\cdot 101 = 2\sum_{r=0}^{n} r \cdot {}^nC_r + \sum_{r=0}^{n} {}^nC_r = 2^{100}\cdot 101$

$2 \cdot n \cdot 2^{n-1} + 2^n = 2^{100}\cdot 101$

$2^n(n+1) = 2^{100}\cdot 101$ $\Rightarrow$ $n = 100$ $\Rightarrow$ $2\left[\frac{n-1}{2}\right] = 98$

30.(55) ${}^nC_7 (2)^{n-7}\left(\frac{1}{3}\right)^7 = {}^nC_8 (2)^{n-8}\left(\frac{1}{3}\right)^8$

${}^nC_7 \cdot 6 = {}^nC_8$ $\Rightarrow$ ${}^nC_7 \cdot 6 = {}^nC_8$ $\Rightarrow$ $\frac{n!}{(n-7)!\,7!}\cdot 6 = \frac{n!}{(x-8)!\,8!}$

$\frac{6}{x-7} = \frac{1}{8}$ $\Rightarrow$ $n - 7 = 48$ $\Rightarrow$ $n = 55$

31.(B) $\left(x^2 + \frac{1}{bx}\right)^{11}$

$T_{r+1} = {}^{11}C_r \cdot (x^2)^{11-r} \cdot \left(\frac{1}{bx}\right)^r = {}^{11}C_r \cdot x^{22-2r} \cdot \frac{1}{b^r \cdot x^r} = {}^{11}C_r \cdot \left(\frac{1}{b}\right)^r \cdot x^{22-3r}$

$22 - 3r = 7 \Rightarrow 3r = 15 \Rightarrow r = 5$

Coefficient of $x^7 = {}^{11}C_5 \cdot \left(\frac{1}{b}\right)^5$

$\left(x - \frac{1}{bx^2}\right)^{11}$; $T_{r+1} = {}^{11}C_r \cdot x^{11-r} \cdot \left(\frac{-1}{bx^2}\right)^r = {}^{11}C_r \cdot \left(\frac{-1}{b}\right)^r \cdot x^{11-3r}$

$11 - 3r = -7 \Rightarrow 3r = 18 \Rightarrow r = 6$

Coefficient of $x^{-7} = {}^{11}C_6 \cdot \left(\frac{-1}{b}\right)^6$

${}^{11}C_5 \left(\frac{1}{b}\right)^5 = {}^{11}C_6 \cdot \left(\frac{-1}{b}\right)^6 \quad \Rightarrow b^6 = b^5 \quad \Rightarrow b = 1 \ ; \ \because b \neq 0$

32.(B) $T_g = {}^{10}c_8 \left(\sqrt{(25^{x-1}+7)}\right)^2 \left((5^{x-1}+1)^{-1/8}\right)^8 = 180 \ = 45(25^{x-1}+7)\,(5^{x-1}+1)^{-1} = 180$

$5^{x-1} = t$

$\frac{t^2+7}{t+1} = 4 \ \Rightarrow \ t = 1, \ t = 3$

$5^{x-1} = 1 \quad \Rightarrow \ x = 1$

33.(B) $\because \quad \sum_{k=0}^{20} \left({}^{20}C_k\right)^2 = \sum_{k=0}^{20} {}^{20}C_k \times {}^{20}C_{20-k}$

$= {}^{40}C_{20}$ [$\because$ Sum of suffix is constant]

34.(49) ${}^9C_0 \cdot {}^{12}C_{12-k} + {}^9C_1 \cdot {}^{12}C_{13-k} + {}^9C_2 \cdot {}^{12}C_{14-k} + \ldots\ldots + {}^9C_9 \cdot {}^{12}C_{21-k}$

${}^9C_0 \cdot {}^{12}C_k + {}^9C_1 \cdot {}^{12}C_{k-1} + {}^9C_2 \cdot {}^{12}C_{k-2} + \ldots\ldots + {}^9C_9 \cdot {}^{12}C_{k-2} = {}^{21}C_k$

Similarly

$\sum {}^8C_0 \cdot {}^{13}C_{13-k+i} = \sum {}^8C_i \cdot {}^{13}C_{k-i} = {}^{21}C_k$

$\therefore \quad A_k = 2 \times {}^{21}C_k$ | $2\left[\frac{21 \times 20 \times 19 \times 18}{1 \cdot 2 \cdot 3 \cdot 4} - \frac{21 \times 20 \times 19}{1 \times 2 \times 3}\right] = 190P$

$A_4 - A_3 = 190P$ | $2 \times \frac{21 \times 20 \times 19}{1 \cdot 2 \cdot 3}\left[\frac{9}{2} - 1\right] = 190P$

$2[{}^{21}C_4 - {}^{21}C_3] = 190P$ | $7 \times 20 \times 19 \times \frac{7}{2} = 190P$

$P = 49$

35.(924) Sum of coefficients = ${}^nc_0 + {}^nc_1 + \_\_\_\_ + {}^nc_n = 2^n = 4096$

$\therefore \ n = 12$

Greatest coefficient $= {}^{12}C_6$

36.(55) $\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$

$T_{r+1} = (-1)^r \cdot {}^{12}C_r \left(\frac{x}{4}\right)^{12-r} \left(\frac{12}{x^2}\right)^r$

$T_{r+1} = (-1)^r \cdot {}^{12}C_r \left(\frac{1}{4}\right)^{12-r} (12)^r \cdot (x)^{12-3r}$

Term independent of $x \Rightarrow 12-3r = 0 \Rightarrow r = 4$

$T_5 = (-1)^4 \cdot {}^{12}C_4 \left(\frac{1}{4}\right)^8 (12)^4 = \frac{3^6}{4^4} \cdot k$

$\Rightarrow \quad k = 55$

37.(15) $3\times(7)^{22} + 2(10)^{22} - 44 = 3\times(48+1)^{11} + 2(9+1)^{22} - 44$

$= 3\times(\textit{multiple of } 48+1) + 2(\textit{multiple of } 9+1) - 44$

$= \textit{multiple of } 18+3+2-44 = \textit{multiple of } 18+5+10-54$

$\Rightarrow \text{Re}\textit{mainder} = 15$

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1.(84) $\dfrac{T_5}{T_{n-3}} = \dfrac{{}^nC_4\left(2^{1/4}\right)^{n-4}\left(3^{-1/4}\right)}{{}^nC_{n-4}\left(2^{1/4}\right)^4\left(3^{-1/4}\right)^{n-4}} = \dfrac{\sqrt[4]{6}}{1}$

$\Rightarrow \quad 2^{\frac{n-8}{4}}3^{\frac{n-8}{4}} = 6^{1/4}$

$\Rightarrow \quad 6^{n-8} = 6$

$\Rightarrow \quad n-8=1 \Rightarrow n=9$

$T_6 = {}^9C_5\left(2^{1/4}\right)^4\left(3^{-1/4}\right)^5 = \dfrac{84}{\sqrt[4]{3}}$

$\therefore \quad \alpha = 84$

2.(D) $3^{2022} = 9^{1011} = (10-1)^{1011} = 10m-1 = 10m-5+4$

$\therefore$ Remainder = 4

3.(A) $(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + ... + x^{500}$

Here, $a = (5+x)^{500}$

$r = \dfrac{x}{5+x}$ $\qquad \boxed{use\ S_n = \dfrac{a(1-r^n)}{1-r}}$

$\text{Sum} = \dfrac{(5+x)^{501} - x^{501}}{(5+x)-x} = \dfrac{(5+x)^{501} - x^{501}}{5}$

Coefficient of x^{101} *in* $\dfrac{(5+x)^{501} - x^{501}}{5}$

4.(83) $\left(2x^3 + \dfrac{3}{x}\right)^{10}$

$T_{r+1} = {}^{10}C_r \cdot \left(2x^3\right)^{10-r} \cdot \left(\dfrac{3}{x}\right)^r$

$= {}^{10}C_r . 2^{10-r} . 3^r . x^{30-3r-r}$

$= {}^{10}C_r . 2^{10-r} . 3^r . x^{30-4r}$

$30 - 4r$ should be even positive no r = 0, 1, 2, 3, ...7

$$\text{Sum} = \sum_{r=0}^{7} {}^{10}C_r .2^{10-r}.3^r = (2+3)^{10} - {}^{10}C_8\, 2^2.3^8 - {}^{10}C_9 2.3^9 - {}^{10}C_{10}3^{10}$$

$$= 5^{10} - 180 \times 3^8 - 20 \times 3^9 - 3^{10}$$

$$= 5^{10} - 3^9 (60 + 20 + 3)$$

$$= 5^{10} - 3^9 \times 83$$

5.(D) $\dfrac{1}{2.3^{10}} + \dfrac{1}{2^2.3^9} + \ldots\ldots\ldots \dfrac{1}{2^{10}.3} = \dfrac{k}{2^{10}.3^{10}}$

$$a = \frac{1}{2.3^{10}};\ r = \frac{1}{2^2.3^9} \times \frac{2.3^{10}}{1} = \frac{3}{2}$$

$$\text{Sum} = \frac{1}{2.3^{10}} \times \left\{ \frac{\left(\frac{3}{2}\right)^{10} - 1}{\frac{3}{2} - 1} \right\} = \frac{1}{2.3^{10}} \left\{ \left(\frac{3}{2}\right)^{10} - 1 \right\} \times \frac{2}{1}$$

$$= \frac{3^{10} - 2^{10}}{2^{10} \times 3^{10}} \quad ; \quad \frac{k}{2^{10}.3^{10}} = \frac{3^{10} - 2^{10}}{2^{10}.3^{10}} \Rightarrow k = 3^{10} - 2^{10}$$

$$k = \left(3^5\right)^2 - \left(2^5\right)^2 = \left(3^5 + 2^5\right)\left(3^5 - 2^5\right)$$

$$= (243 + 32)(243 - 32) \quad ; \quad = (275)(211)$$

$$\frac{k}{6} = \frac{(5)(1)}{6} \Rightarrow k = 5$$

6.(286)

$${}^{10}C_1 + 3.2{}^{10}C_2 + 5.3{}^{10}C_3 + \ldots. \text{ up to } 10 \text{ terms}$$

$$= \sum_{r=1}^{10} (2r-1)r\ {}^{10}C_r = \sum_{r=1}^{10} \left(2(r-1)+1\right)r.\frac{10}{r}\,{}^{9}C_{r-1}$$

$$= 10\left[\sum_{r=2}^{10} 2(r-1).\frac{9}{r-1}.{}^{8}C_{r-2} + \sum_{r=1}^{10} {}^{9}C_{r-1}\right] = 100 \times 2^9$$

Also, $(1+x)^{10} = C_0 + C_1x^1 + C_2x^2 + \ldots\ldots + C_{10}x^{10}$

$$\int_{x=0}^{x=1} (1+x)^{10}\,dx = \left[C_0x + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + \ldots\ldots\ldots \frac{C_{10}x^{11}}{11}\right]_{x=0}^{x=1}$$

$$\frac{2^{11}-1}{11} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots\ldots + \frac{C_{10}}{11}$$

$\therefore \quad C_1 + 3.2C_2 + 5.3C_3 + \ldots\ldots$ up to 10 terms

$$= \frac{\alpha \times 2^{11}}{2^{\beta}-1}\left(C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots\ldots \text{upto } 10 \text{ terms}\right)$$

$$\Rightarrow \qquad 100 \times 2^9 = \frac{\alpha \times 2^{11}}{2^{\beta}-1}\left(\frac{2^{11}-1}{11}\right) \Rightarrow 275 = \frac{\alpha\left(2^{11}-1\right)}{2^{\beta}-1}$$

$$\Rightarrow \qquad \alpha = 275,\ \beta = 11 \Rightarrow \alpha + \beta = 286$$

7.(4) $\alpha_n = 19^n - 12^n$

$$\frac{31\alpha_9 - \alpha_{10}}{57\alpha_8} = \frac{31\left(19^9 - 12^9\right) - \left(19^{10} - 12^{10}\right)}{57\left(19^8 - 12^8\right)}$$

$$= \frac{19^9\left(31-19\right) - 12^9\left(31-12\right)}{57\left(19^8 - 12^8\right)} = \frac{12 \times 19\left[19^8 - 12^8\right]}{57\left[19^8 - 12^8\right]} = 4$$

8.(C) $A = \sum_{n=1}^{\infty} \frac{1}{(3+(-1)^n)^n}$ and $B = \sum_{n=1}^{\infty} \frac{(-1)^n}{(3+(-1)^n)^n}$

$$A = \frac{1}{2} + \frac{1}{4^2} + \frac{1}{2^3} + \frac{1}{4^4} + \frac{1}{2^5} + \frac{1}{4^6} + \ldots$$

$$B = \frac{-1}{2} + \frac{1}{4^2} - \frac{1}{2^3} + \frac{1}{4^4} - \frac{1}{2^5} + \frac{1}{4^6} + \ldots$$

$$A = \frac{\frac{1}{2}}{1-\frac{1}{2^2}} + \frac{\frac{1}{4^2}}{1-\frac{1}{4^2}} = \frac{2}{3} + \frac{1}{15} = \frac{11}{15}$$

$$B = -\frac{2}{3} + \frac{1}{15} = -\frac{9}{15} \qquad \Rightarrow \qquad \frac{A}{B} = -\frac{11}{9}$$

9.(102) ${}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + \ldots + {}^{60}C_{20} = \frac{m}{n}\,{}^{60}C_{20}$

$$\underbrace{{}^{41}C_0 + {}^{41}C_1} + {}^{42}C_2 + \ldots + {}^{60}C_{20}$$

$$\underbrace{{}^{42}C_1 + {}^{42}C_2} + \ldots + {}^{60}C_{20}$$

$$\underbrace{{}^{43}C_2 + \ldots\ {}^{60}C_{20}}$$

$${}^{61}C_{20} = m/n\ {}^{60}C_{20}$$

$$\frac{61!}{41!\,20!} = \frac{m}{n}\,\frac{60!}{40!\,20!}$$

$$\frac{m}{n} = \frac{61}{41}$$

$$m + n = 102$$

10.(C) $(2021)^{2023} = (289\times 7-2)^{2023}$

$= 7\lambda - 2^{2023} = 7\lambda - 2(2^3)^{674}$

$= 7\lambda - 2(7+1)^{674} = 7\lambda - 2(7\mu+1) = 7(\lambda-2\mu)-2 = 7(\lambda-2\mu-1)+5$

Where λ *and* μ are integer so remainder is "5"

11.(57) coefficients and there cumulative sum are:

Coefficient	Commulative sum
$x^{7n} \to {}^7C_0$	1
$x^{6n-5} \to 2.{}^7C_1$	$1+14$
$x^{5n-10} \to 2^2.{}^7C_2$	$1+14+84$
$x^{4n-15} \to 2^3.{}^7C_3$	$1+14+84+280$
$x^{3n-20} \to 2^4.{}^7C_4$	$1+4+84+280+560=939$
$x^{2n-25} \to 2^5.{}^7C_5$	

$3n-20 \ge 0 \cap 2n-25 < 0 \cap n \in I$

$\therefore \quad 7 \le n \le 12$

Sum = $7+8+9+10+11+12 = 57$

12.(C) $n = 7$; success = p ; $q = 1-p$

$p(x=3) = 5p(x=4)$

${}^7C_3 \,.\, p^3(1-p)^4 = 5\times {}^7C_4 \,.\, p^4 \,.\, (1-p)^3$

$p = \frac{1}{6};\ q = \frac{5}{6}$

Mean = $np = 7\times\frac{1}{6}$; Variance $= npq = 7.\frac{1}{6}.\frac{5}{6}$

Mean + Variance $= \frac{77}{36}$

13.(5) $T_{r+1} = {}^{60}C_r \left(\frac{\sqrt{x}}{5^{1/4}}\right)^{60-r}\left(\frac{\sqrt{5}}{x^{1/3}}\right)^r$

$x^{30-\frac{r}{2}-\frac{r}{3}} = x^{10}$

$20 = \frac{5r}{6} \Rightarrow r = 24$

$T_{25} = {}^{60}C_{24}\times x^{16}\times\frac{5^{12}}{5^9}$

Coeff $= {}^{60}C_{24}\times 5^3$

$$\Rightarrow \frac{\overset{①}{60}\times\ldots\times\overset{①}{55}\times\ldots\times\overset{②}{40}\times\ldots\times\overset{①}{45}\times\ldots\times\overset{①}{40}\times\ldots\times 5^3}{24\times\ldots\times\underset{①}{20}\times\ldots\times\underset{①}{15}\times\ldots\times\underset{①}{10}\times\ldots\times\underset{①}{5}\times\ldots}$$

$= 5^5\times I$

$K = 5$

14.(B) $\left(1-x^2+3x^3\right)\left({}^{11}C_r\left(\frac{5}{2}x^3\right)^{11-r}\left(\frac{-1}{5x^2}\right)^r\right)$

$\left(1-x^2+3x^3\right)\left({}^{11}C_r\left(\frac{5}{2}\right)^{11-r}\left(-\frac{1}{5}\right)^r x^{33-3r}x^{-2r}\right)$

$\left(1-x^2+3x^3\right)\left({}^{11}C_r\left(\frac{5}{2}\right)^{11-r}\left(-\frac{1}{5}\right)^r x^{33-5r}\right).$

${}^{11}C_r\left(\frac{5}{2}\right)^{11-r}\left(-\frac{1}{5}\right)^r\left(x^{33-5r}-x^{35-5r}+3x^{36-5r}\right)$

Clearly $r=7.$ for independent term.

$\therefore \quad {}^{11}C_7\left(\frac{5}{2}\right)^4\left(-\frac{1}{5}\right)^7=\frac{33}{200}$

15.(A) $\sum_{k=1}^{31}{}^{31}C_k.{}^{31}C_{32-k}-\sum_{k=1}^{30}{}^{30}C_k.{}^{30}C_{31-k}$

${}^{62}C_{32}-{}^{60}C_{31}$

$\frac{62!}{32!30!}-\frac{60!}{31!29!}=\frac{60!}{31!29!}\left(\frac{62\times61}{32\times30}-1\right)=\frac{60!}{31!29!}\left(\frac{3782-960}{32\times30}\right)=\frac{60!}{(31)!\times(30)!}.\frac{2822}{32}$

Here $\alpha=\frac{2822}{32}$ So $16\alpha=\frac{2822}{32}\times16=1411$

Correction option A.

16.(2) General term in the expansion of $\left(2x^3+\frac{3}{x^k}\right)^{12}$ is

$T_{r+1}={}^{12}C_r\left(2x^3\right)^{12-r}\left(\frac{3}{x^k}\right)^r={}^{12}C_r\left(2\right)^{12-r}\left(3\right)^r.x^{36-3r-rk}$

If $36-3r-rk=0$

$k=\frac{36}{r}-3 \Rightarrow r=1,2,3,4,6,9,12$

For $2^8\times$ odd integer $\Rightarrow r=4$ and 6 are the only suitable values.

17.(C) $q^n=(1+8)^n=1+8n+8^2({}^nC_2+{}^nC_3 8.....{}^nC_n 8^{n-2})$

$\Rightarrow \alpha={}^nC_2+{}^nC_3 8....{}^nC_n 8^{n-2}$

$6^n=(1+5)^n=1+5n+5^2({}^nC_2+{}^nC_3 5....{}^nC_n 5^{n-2})$

$\Rightarrow \beta={}^nC_2+{}^nC_3 5....{}^nC_n 5^{n-2}$

$\Rightarrow \alpha-\beta={}^nC_3(8-5)+{}^nC_4(8^2-5^2)\ .....{}^nC_n(8^{n-2}-5^{n-2})$

18.(5) $T_{r+1} = {}^{15}C_r (2x^{1/5})^{15-r} \left(-x^{-1/5}\right)^r$

$= {}^{15}C_r \, 2^{15-r} \cdot (-1)^r \cdot x^{\left(3-\frac{2r}{5}\right)}$

Put $r = 10$ coeff. of $x^{-1} = {}^{15}C_{10} 2^5 = m$

Put $r = 15$ coeff. of $x^{-3} = {}^{15}C_{15} 2^0 (-1)^{15} = -{}^{15}C_{15} = -1 = n$

$mn^2 = {}^{15}C_r \cdot 2^r \Rightarrow \quad {}^{15}C_{10} 2^5 = {}^{15}C_r \cdot 2^r$

$\Rightarrow \quad {}^{15}C_5 \cdot 2^5 = {}^{15}C_r \cdot 2^r \quad \Rightarrow \quad r = 5$

19.(D) General term in expansion f $\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10}$

is $\frac{10!}{a!b!c!} 3^a (-2)^6 5^c x^{3a+2b-5c}$ $a, b, c \geq 0$ and $a + b + c = 10$

For constant term $3a + 2b = 5c \; = 5(10 - (a+b))$

$\Rightarrow 8a + 7b = 50 \;\; a, b \geq 0 \; and \; a + b \leq 10$

$7b = 50 - 8a$

$\Rightarrow 50 - 8a \geq 0$ and is divisible by 7

Only possibility $a = 1, b = 6, c = 3$

$\Rightarrow$ Constant term $= \frac{10!}{6!3!} 3 \times 2^6 \times 5^3 = 2^9 \times 3^2 \times 5^4 \times 7 \Rightarrow k = 9$

20.(D) $\text{Re}\left[(11)^{1011} + (1011)^{11}\right]$

$\text{Re}\left[(9+2)^{1011} + (112 \times 9 + 3)^{11}\right]$

$\text{Re}\left[2^{1011} + 3^{11}\right]$

$\text{Re}\left[(9-1)^{337} + \frac{3^{11}}{9}\right] = -1 + 0$

Hence remainder $-1 + 9 = 8$

21.(A) $(2021)^{2022} + (2022)^{2021}$

$(2023-2)^{2022} + (2023-1)^{2021}$

$(1-2023)^{2022} - (1-2023)^{2021}$

$[{}^{2022}C_0 \; 2^{2022} - {}^{2022}C_1 \; 2^{2021} (2023)^1 + \ldots\ldots]$

$-[{}^{2021}C_0 - {}^{2021}C_1 \; (2023)^1 + \ldots\ldots]$

$= (2^{2022} + 7k_1) - (1 + 7k_2) = (2^3)^{674} - 1 + 7(k_1 - k_2) = (1+7)^{674} - 1 + 7(k_1 - k_2)$

$= 1 + 7k_3 - 3 + 7(k_1 - k_2) = 7k$ Remainder = 0

22.(C) $7^{2022}+3^{2022}=(5+2)^{2022}+(5-2)^{2022}$

$=\left(5\lambda_1+2^{2022}\right)+5\lambda_2+(-2)^{2022}$

$=5(\lambda_1+\lambda_2)+2^1.2^{2022}=5(\lambda_1+\lambda_2)+2(4)^{1011}$

$=5(\lambda_1+\lambda_2)+2\left[(5-1)^{1011}\right]=5(\lambda_1+\lambda_2)+2\left[5\lambda_3+(-1)^{1011}\right]$

$=5(\lambda_1+\lambda_2)+10\lambda_3-2$

Hence, remainder = –2 i.e.,3

23.(6006) $\left(t^2x^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$

$T_{r+1}={}^{15}C_r\left(t^2x^{\frac{1}{5}}\right)^{15-r}\cdot\frac{(1-x)^{\frac{r}{10}}}{t^r}$

For independent of t,

$30-2r-r=0 \quad \Rightarrow r=10$

So, maximum value of ${}^{15}C_{10}x(1-x)$ will be at $x=\frac{1}{2}$ i.e. 6006

24.(57) ${}^4C_2\left(\frac{1}{\sqrt{6}}\right)^2\beta^2,\ {}^2C_1(-3\beta),\ {}^6C_3\left(\frac{-\beta}{2}\right)^3$

$\beta^2,-6\beta,\frac{-5}{2}\beta^3$

$-12=\beta-\frac{5}{2}\beta^2\Rightarrow 5\beta^2-2\beta-24=0\Rightarrow\beta=\frac{12}{5}$

25.(99) $1+(2+2^{49}-1)(2^{49}-1)=2^{98}$

26.(B) $\sum_{r=1}^{20}\left(r^2+1\right)\underline{|r}$

$\sum_{r=1}^{20}\left((r+1)^2-2r\right)\underline{|r}$

$\sum_{r=1}^{20}(r+1)\underline{|r+1}-r\underline{|r}-\sum_{r=1}^{20}r\underline{|r}$

$\sum_{r-1}^{20}(r+1)\underline{|(r+1)}-r\underline{|r}-\sum_{r=1}^{20}\left(\underline{|r+1}-\underline{|r}\right)$

$=\left(21\underline{|21}-1\right)-\left(\underline{|21}-1\right)$

$=20\underline{|21}=\underline{|22}-2-\underline{|21}$

27.(221) $\sum_{k=1}^{10} K^2 \left(10_{C_K}\right)^2 = 22000L$

$\sum_{k=1}^{10} \left(10 \cdot 9_{C_{K-1}}\right)^2 = 22000L$

$\therefore \quad \sum_{r=0}^{n} \left(n_{C_r}\right)^2 = 2n_{C_n}$

So, $18_{C_9} = 220L$

$48620 = 220L$

$\boxed{L = 221}$

28.(A) $\Rightarrow {}^nC_0\left({}^nC_1 + {}^nC_2 + ... + {}^nC_n\right) + {}^nC_1\left({}^nC_0 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n\right)$

$+ ..... + {}^nC_n\left({}^nC_0 + {}^nC_1 + ... + {}^nC_{n-1}\right)$

$\Rightarrow {}^nC_0\left(2^n - 1\right) + {}^nC_1\left(2^n - {}^nC_1\right) + ... + {}^nC_n\left(2^n - {}^nC_n\right)$

$= 2^n\left\{{}^nC_0 + {}^nC_1 + ... + {}^nC_n\right\} - \left(\left({}^nC_0\right)^2 + \left({}^nC_1\right)^2 + ... + \left({}^nC_n\right)^2\right) = 2^n \times 2^n - {}^{2n}C_n = 2^{2n} - {}^{2n}C_n$

29.(24) $\frac{n+1}{1+\left|\frac{a}{bx}\right|} - 1 < r < \frac{n+1}{1+\left|\frac{a}{bx}\right|} \left(\frac{a}{bx} = \frac{3}{6 \times \frac{3}{2}} = \frac{1}{3}\right)$

$\Rightarrow \frac{n+1}{1+\frac{1}{3}} - 1 < 8 < \frac{n+1}{1+\frac{1}{3}}$

$\Rightarrow \frac{29}{3} < n < \frac{33}{3} \Rightarrow n = 10$

Now $k = \frac{{}^nC_6 3^{n-6}\left(6x\right)^6}{{}^nC_3 3^{n-3}\left(6x\right)^3} = 14$

$\Rightarrow k + n = 24$

30.(23) Since coefficient of x is – 3

$\Rightarrow {}^pC_1 - {}^qC_1 = -3$

$\Rightarrow \; p - q = -3 \qquad ...(1)$

Comparing coefficients of x^2

$- {}^pC_1 \, {}^qC_1 + {}^pC_2 + {}^qC_2 = -5$

$-pq + \frac{p\left(p-1\right)}{2} + \frac{q\left(q-1\right)}{2} = -5 \qquad ...(2)$

Solving (1) and (2) $p = 8, q = 11$

Coefficient of x^3 is $-{}^qC_3 + {}^pC_3 + {}^pC_1 \, {}^qC_2 - {}^pC_2 \, {}^qC_1$

$= - {}^{11}C_3 + {}^8C_3 + {}^8C_1 \, {}^{11}C_2 - {}^8C_2 \, {}^{11}C_1 = 23$

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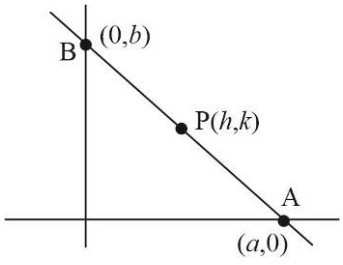
1.(D) Equation of AB = Path of man $\ddot{Y}\ \frac{x}{a}\ \frac{y}{b}\ 1$



Since it passes through (h, k)

$?\ \frac{h}{a}\ \frac{k}{b}\ 1$ ...(i)

Also, $\frac{\frac{1}{a}\ \frac{1}{b}}{2}\ \frac{1}{4}$

$?\ \frac{1}{a}\ \frac{1}{b}\ \frac{1}{2}$ ...(ii)

? AB passes through of a fixed point (2, 2)

From (i) & (ii)

2.(B) $\frac{x\ 3}{1}\ \frac{y\ 5}{1}\ 2\frac{3\ 5\ 1}{2}$ [Finding image of point (3, 5)]

$x\ 4\qquad y\ 4$

Clearly (4, 4) lies one $x\ 2^{2}\ y\ 4^{2}\ 4$

3.(C) P.O.I of the given lines are

$A\ 3, 0\ , B\ 1, 1\ , C\ 2, 2$

$AB\ \sqrt{5}, AC\ \sqrt{5}, BC\ \sqrt{2}$ Isosceles

4.(C) Slope of the given line $\frac{m\ r\tan D}{1\ Bm\tan D}$

Where $m\ 3\sqrt{2}$ and $\tan D\ \sqrt{2}$

Slope $\frac{3\sqrt{2}\ r\sqrt{2}}{1\ B6}\ \frac{4\sqrt{2}}{5}$ or $\frac{2\sqrt{2}}{7}$

Equation of the line $y\ 3\ \frac{4\sqrt{2}}{5}(x\ 1)$

$y\ 3\ \frac{2\sqrt{2}}{7}(x\ 1)$

One of the lines will be $5y\ 15\ 4\sqrt{2}x\ 4\sqrt{2}\ \ddot{Y}\ 4\sqrt{2}x\ 5y\ (15\ 4\sqrt{2})\ 0$

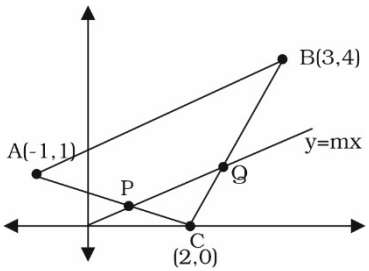
5.(D) 3x + 4y = 9 and y = mx + 1

$3x + 4(mx + 1) = 9$ Ÿ $x(3 \quad 4m) \quad 5$ Ÿ $x \quad \frac{5}{3 \quad 4m}$

For x to be an integer (3 + 4m) must be a factor of 5

Ÿ $3 \quad 4m \quad r1$ or $r\,5$ Ÿ $m \quad 2$ or $m \quad 1$

6.(A)



Equation of side AC $x \quad 3y \quad 2$

Point P $y \quad mx$

$x_1 \quad \frac{2}{1 \quad 3m}$ ... (i)

Equation of side BC $4x \quad y \quad 8$

Point Q $y \quad mx \quad x_2 \quad \frac{8}{4 \quad m}$ ... (ii)

$A_1 \quad \frac{1}{2}\begin{vmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 3 & 4 & 1 \end{vmatrix} \quad \frac{13}{2}$

$A_2 \quad \frac{1}{2}\begin{vmatrix} 2 & 0 & 1 \\ x_1 & mx_1 & 1 \\ x_2 & mx_2 & 1 \end{vmatrix} \quad \frac{1}{2}\,u2m\,|x_1 \quad x_2| \quad 2\left|\frac{2}{1 \quad 3m} \quad \frac{8}{4 \quad m}\right| \quad \frac{26m^2}{1 \quad 3m \quad 4 \quad m}$

$A_1 \quad 3A_2$ Ÿ $\frac{13}{2} \quad 3.\frac{26m^2}{1 \quad 3m \quad 4 \quad m}$

$1 \quad 3m \quad 4 \quad m \quad 12m^2$; $15m^2 \quad 11m \quad 4 \quad 0$

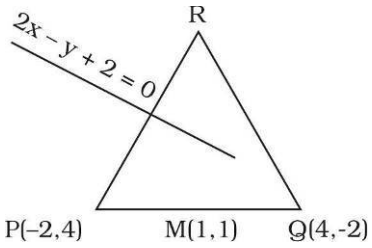
$m \quad 1 \quad 15m \quad 4 \quad 0$; $m \quad 1, m \quad \frac{4}{15}$; $m \quad 1$

7.(B) Right Bisector of PQ

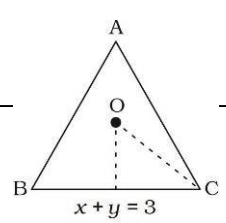
$y \quad x$

Solving $2x \quad y \quad 2 \quad 0$ and $y \quad x$

$2, \quad 2$ is centre



8.(C) $r \quad \frac{|0 \quad 0 \quad 3|}{\sqrt{2}}$



r $\frac{3}{\sqrt{2}}$

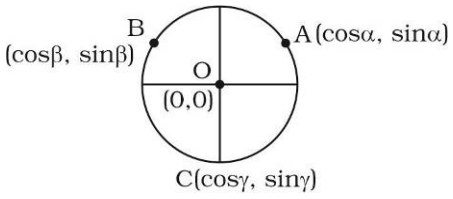
R OC 2r

' $\sin 30^\circ = \frac{r}{R} = \frac{1}{2}$

R r 3r $\frac{9}{\sqrt{2}}$

9.(144) m_{OA} tan D, m_{OB} tan E m_D tan

Circumcentre { 0(0,0)



Orthocentre is (cos D cos E cos , sin D sin E sin)

But orthocentre lies on y-axis

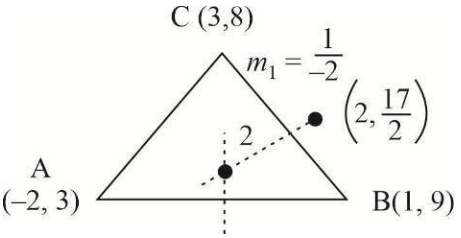
? cos D cos E cos J 0

$\cos^3 D \cos^3 E \cos^3 J = 3\cos \cos \cos = \cos E$

$\frac{\cos 3D + 3\cos D}{4} + \frac{\cos 3E + 3\cos E}{4} + \frac{\cos 3J + 3\cos J}{4} = 3\cos D\cos E\cos J$

Ÿ $\cos 3D + \cos 3E + \cos 3J = 12\cos \cos \cos$ Ÿ $\left(\frac{\cos 3D + \cos 3E + \cos 3J}{\cos D\cos E\cos J}\right)^2 = (12)^2 = 144$

10.(9)



Eq. of line y $\frac{17}{2}$ 2 x 2

Put x = 0

y $\frac{17}{2}$ 4 Ÿ 2y 17 8

2y 9

$\frac{D}{2} = \frac{9}{2}$ Ÿ D $\frac{18}{2}$ 9

11.(A) For the pair of lines ax^2 2hxy by^2 0, the equation of angle bisectors is

$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$

So for the lines: x^2 4xy $5y^2$ 0

$$\frac{x^2 \quad y^2}{6} \quad \frac{xy}{2}$$

$$x^2 \quad 3xy \quad y^2 \quad 0$$

12.(B) $(a,b) \xrightarrow{a}$ (b,a)

$(b,a) \xrightarrow{b}$ $(b \quad 2,a)$

$(b \quad 2,a) \xrightarrow{c}$ $(b \quad 2 \quad i) \quad \left(\frac{1 \quad i}{\sqrt{2}}, \frac{b \quad 2 \quad a}{\sqrt{2}} \quad i \quad \frac{b \quad 2 \quad a}{\sqrt{2}}\right)$

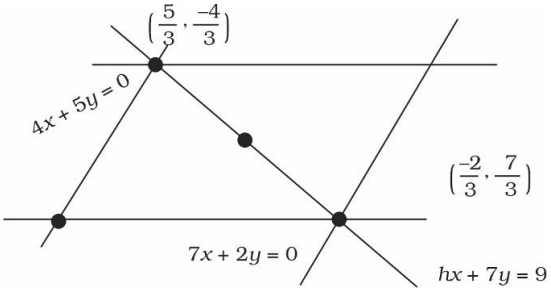
$$\frac{b \quad a \quad 2}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}$$

$$b \quad a \quad 3$$

$$\frac{b \quad a \quad 2}{\sqrt{2}} \quad \frac{7}{\sqrt{2}} \quad b \quad a \quad 5$$

$$\frac{b \quad 1}{a \quad 4} \quad 2a \quad b \quad 9$$

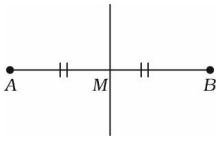
13.(B) Mid point of diagonal $\left(\frac{1}{2}, \frac{1}{2}\right)$



Other diagonal $y \quad x$

14.(A) $A\ 0,6$ and $B\ 2t,0$

Co-ordinates of $M\ \{\ t,3$



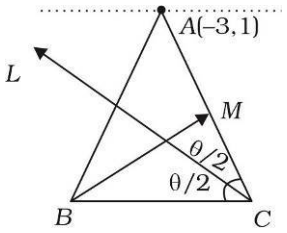
Equation of perpendicular bisector of AB is

$y \quad 3 \quad \frac{t}{3} x \quad t \quad ?$ Co-ordinates of $C\ \{\ \left(0,3 \quad \frac{t^2}{3}\right)$

Let $P\ h,k$ be the mid-point of MC

$? \quad h \quad \frac{t}{2}, k \quad 3 \quad \frac{t^2}{6}$ Ÿ $2x^2 \quad 3y \quad 9 \quad 0$ (By eliminating t)

15.(C) BM is the median. L is angle bisector from C.



BM : 2x y 3 0

L : 7x 4y 1 0

Let M D, E Ÿ C 2 D3,2 E (using mid-point formula)

Now, M D, E satisfies BM Ÿ 2 D E 3 0 .....(i)

& C 2 D 3,2 E1 satisfies L Ÿ 7 2 D 3 4 2 E1 1 0

Ÿ 14 D 8 E 24 0 .....(ii)

(i) & (ii) Ÿ D 0; E 2 ŸM 0,3 & C 3,5

Now; m_{AC} $\frac{4}{6}$ $\frac{2}{3}$ & m_L $\frac{7}{4}$? tan $\frac{T}{2}$ $\left|\frac{m_{AC} \quad m_L}{1 \quad m_{AC} \quad m_L}\right|$ $\frac{1}{2}$

Now; tan T $\frac{2\tan T/2}{1 \quad \tan^2 T/2}$ 4 / 3

16.(6) Solving x y 1 0

x 2y 3 0

2x 5y 11 0

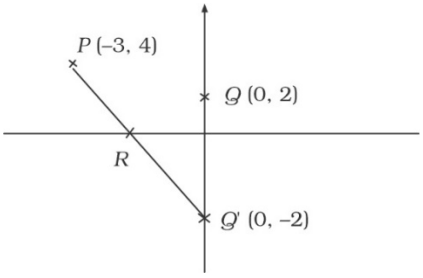
We get P (1, 2)

Q (2, 3)

R (7, 5)

Area of 'PQR $\frac{3}{2}$? area of 'ABC $\frac{3}{2}$ u4 6

17.(1250)



For PR RQ to be minimum P, R, Q' Should be collinear equation of PQ'

y 2 2x

y 0 Ÿ x 1 ? R(1, 0)

PR^2 20

RQ^2 5

18.(C) L_1 : x cosec *D* y sec *D* k cot 2 *D*k $\frac{\cos 2D}{\sin 2D}$

$2x\cos D \;\; 2y\sin \;\; D\, k\cos 2$

$L_1 : x\cos D \;\; y\sin \;\; D\,\frac{k}{2}\cos 2 \;\; D\,0$

Dist. From $(0,0)$ $\left|\frac{0 \;\; 0 \;\; \frac{k}{2}\cos 2\, D}{\sqrt{\sin^2 D \;\; \cos^2 \;\; D}}\right|$ P

$P^2 \;\; \frac{k^2\cos^2 2\, D}{4}$

Similarly q $\left|\frac{0 \;\; 0 \;\; k\sin 2\, D}{\sqrt{\sin^2 D \;\; \cos^2 \;\; D}}\right|$ Ÿ $q^2 \;\; k^2\sin^2 2\, D$

$4P^2 \;\; q^2 \;\; k^2(\sin^2 2\, D \;\; \cos^2 2\;) D \;\; k^2$

19.(C) Area of ' $|\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ b & 2b & 1 \;\; 1 \\ 0 & b & 1 \end{vmatrix}| \;\; 1$

$\begin{vmatrix} a & 0 & 1 \\ b & 2b & 1 \;\; 1 \\ 0 & b & 1 \end{vmatrix} \;\; r2 \qquad$ Ÿ $a \;\; 2b \;\; 1 \;\; b \;\; 0 \;\; b^2 \;\; 0 \;\; r2$

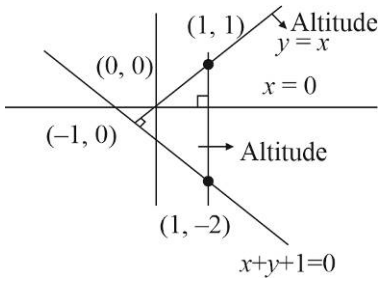
$a \;\; \frac{r2 \;\; b^2}{b \;\; 1}.$

$a \;\; \frac{2 \;\; b^2}{b \;\; 1}$ and $a \;\; \frac{2 \;\; b^2}{b \;\; 1}$

Sum of possible values of 'a' is $\frac{2b^2}{a \;\; 1}$

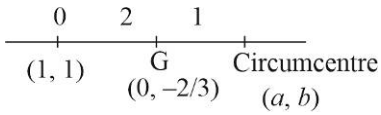
JEE Advanced 2021

1.(B)



Angular point (0, 0), (–1, 0), (1, –2)

Centroid $\left(0, -\frac{2}{3}\right)$



a $-\frac{1}{2}$, b $-\frac{3}{2}$; $x^2 + y^2 + x + 3y = 0$

Solutions 2 & 3

$\left(\frac{h\sqrt{2} + k + 1}{\sqrt{3}}, \frac{h\sqrt{2} - k - 1}{\sqrt{3}}\right)$, r^2

$|2h^2 - (k+1)^2| = 3r^2$

$(k+1)^2 - 2h^2 = 3r^2$

Locus of P $y = 1 + 2x$, $|(y-1)^2 - 2x^2| = 3r^2$

$2x^2 = 3r^2$; $x^2 = \frac{3}{2}r^2$; $x = \sqrt{\frac{3}{2}}$

Distance between R, S $\sqrt{5}|x_1 - x_2| = \sqrt{270}$

2.(9) $r^2 = 9$

Now locus is $|(y-1) - 2x^2| = 27$

$R\left(-3\sqrt{\frac{3}{2}}, 3\sqrt{6} + 1\right)$, $S\left(3\sqrt{\frac{3}{2}}, -3\sqrt{6} + 1\right)$

Mid-point (0, 1)

Perpendicular bisector $x + 2y = 2$

Intersect $|(y-1)^2 - 2x^2| = 27$ at R 'S'

$7(y-1)^2 = 27$; $(y-1) = \pm\sqrt{\frac{27}{7}}$ Ÿ $|y_1 - y_2| = 2\sqrt{\frac{27}{7}}$

3.(77.14) $d^2 = \frac{20 \times 27}{7} = \frac{540}{7} = 77.143 = 77.14$

Solutions of Archive - JEE Main & Advanced

Straight Line	Class - XI | Mathematics

JEE Main 2022

1.(C) $\frac{1}{2}\begin{vmatrix} D & 0 & 1 \\ 1 & D & 1 \\ 0 & D & 1 \end{vmatrix}$ 4 Ÿ D 8

Now given points (8, 8), (8,8), (64, E)or (8,8),(8, 8),(64, E)are collinear

? $\begin{vmatrix} 8 & 8 & 1 \\ 8 & 8 & 1 \\ 64 & E & 1 \end{vmatrix}$ 0 Ÿ E 64

2.(51)

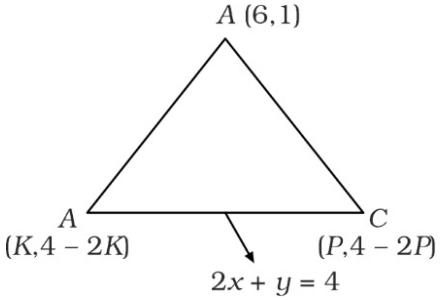
z 0, $\frac{x}{\frac{1}{8}} \quad \frac{y}{\frac{1}{4\sqrt{2}}}$ 1

y 0, $\frac{x}{\frac{1}{8}} \quad \frac{y}{\frac{1}{6\sqrt{3}}}$ 1

If z 0, $\frac{x}{a} \quad \frac{y}{b}$ 1 & y 0, $\frac{x}{a} \quad \frac{z}{c}$ 1

(S.D) ^{2} $\frac{\frac{1}{a^2} \quad \frac{1}{b^2} \quad \frac{1}{c^2}}{4} \quad \frac{64 \quad 32 \quad 108}{4} \quad \frac{204}{4}$ 51

3.(C)



? AB AC

6 K ^{2} 3 2K ^{2} 6 P ^{2} 3 2P ^{2}

P K 5 P K 24 0

? P zK so $\boxed{5\ P\ K\ \ 24}$

Centroid

$$D\ \frac{K\ \ P\ \ 6}{3},\ E\ \frac{9\ \ 2\ P\ \ K}{3}$$

$$D\ \ E\ \frac{15\ \ P\ \ K}{3}\ \ \frac{15\ \ \frac{24}{5}}{3}$$

$$D\ \ E\ \frac{17}{5}$$

15 D E 51

4.(B) Points A lies on L_2

$$A\ \left(D, 4\ \ \frac{4}{3}\ D\right)$$

Points B lies on L_1

Points P divides AB internally in the ratio 1 : 3

$$\text{Ÿ}\ P\ 2,3\quad P\left(\frac{2\ D\ \ E}{4}, \frac{\left(4\ \ \frac{5}{3}\ D\right)\ 3\ \ \left(2\ \ \frac{2}{5}\ E\right)\ 1}{4}\right)$$

$$\text{Ÿ}\ D\ \frac{3}{13},\ E\ \frac{95}{13}$$

Point $A\left(\frac{3}{13}, \frac{56}{13}\right)$, $B\left(\frac{95}{13}, \frac{12}{13}\right)$

Vertex C of triangle is the point of intersection of L_1 & L_2

$$\text{Ÿ}\ C\left(\frac{15}{13}, \frac{32}{13}\right)$$

$$\text{Area}\ \ 'ABC\ \ \frac{1}{2}\left\|\begin{matrix}\frac{3}{13} & \frac{56}{13} & 1\\ \frac{95}{13} & \frac{12}{13} & 1\\ \frac{15}{13} & \frac{32}{13} & 1\end{matrix}\right\|\ \ \frac{1}{2\ u\ 13^3}\left\|\begin{matrix}3 & 56 & 13\\ 95 & 12 & 13\\ 15 & 32 & 13\end{matrix}\right\|$$

area $'ABC\ \ \frac{132}{13}$ sq. units.

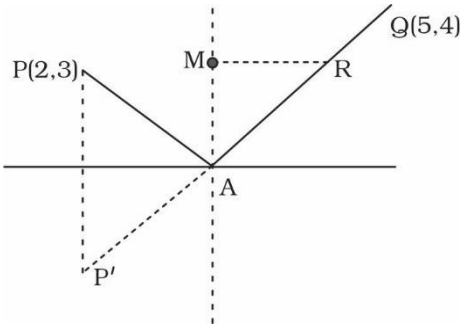
5.(31) p' mirror image of P about x axis.

Ÿ p' 2, 3

Now equation of line P'Q is 7x 3y 23

Point A is $\left(\frac{23}{7}, 0\right)$

Now point R divides the portion AQ in the ratio of 2 : 1



Ÿ Co-ordinate of R $\left(\frac{31}{7}, \frac{8}{3}\right)$

Ÿ Co-ordinate of point M $\left(\frac{23}{7}, \frac{8}{3}\right)$

Ÿ 7 D 3 E 23 8 31

6.(C) Let the given ' have vertices A, B, C

Let AB have equation x 2y 1 0

And AC have equation 2x y 1 0

Ÿ A : (1, 1)

Let H : $\left(\frac{7}{3}, \frac{7}{3}\right)$

Ÿ Slope of BC 1{AH ABC}

Also, altitude through B has slope $\frac{1}{2}$

Ÿ Equation of BH : x 2y 7

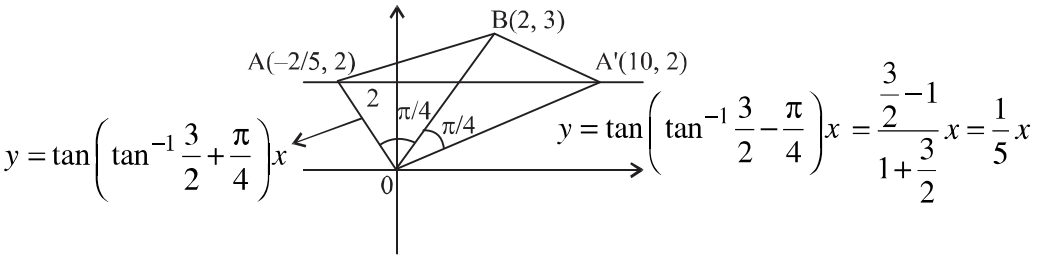
Solving with equation of AB : B(3, 2)

Ÿ C lies on 2x y 1 and $\frac{x \quad y \quad 5}{\text{Eq. of BC}}$

Ÿ C(2, 3)

Ÿ Centroid: (2, 2)

7.(C)



$\frac{\frac{3}{2} \quad 1}{1 \quad \frac{3}{2}}$ x 5x

AA' 10 $\frac{2}{5}$ $\frac{52}{5}$

8.(D) $\left|\frac{3D\ 4\ E12}{5}\right|\ 1$

$\left|\frac{8D\ 6\ E11}{10}\right|\ 1$

Now solving

$3D\ 4\ E\ 7$

$8D\ 6\ E\ 1$

$D\ \ \frac{23}{25}\ \ E\frac{106}{100}$

These point satisfy the given conduction

Hence $100(D\ \ )E\ \ 100\left(\frac{23}{25}\ \ \frac{106}{100}\right)\ 14$

9.(C) Let $B(x_1,x_1\ \ 2)$

$\sqrt{(x_1\ \ 4)^2\ \ (x_1\ \ 2\ \ 3)^2}\ \ \frac{\sqrt{29}}{3}$

Squaring on both side

$18x_1^2\ \ 162x_1\ \ 340\ \ 0$

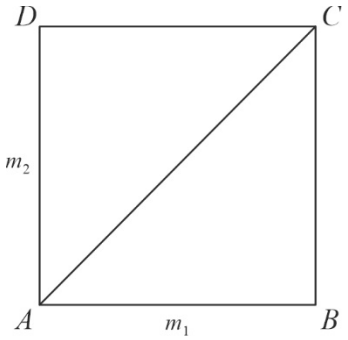
$x_1\ \ \frac{51}{9}$ or $x_1\ \ \frac{10}{3}$

$y_1\ \ \frac{33}{9}$ or $y_1\ \ \frac{4}{3}$

Option (C) will satisfy $\left(\frac{10}{3},\frac{4}{3}\right)$

10.(B) $m_1.m_2\ \ 1$

$a^2\ \ 11a\ \ 3\left(m_1^2\ \ \frac{1}{m_1^2}\right)\ 220$



AC : $\cos D\ \sin Dx\ \ \sin D\cos yD\ 10$

$m_{AC}\ \ \frac{\sin D\ \cos D}{\sin D\ \cos D}\ \tan T$

Equation of line making ' $\frac{S}{4}$ with AC :

$m_1, m_2 \quad \frac{m \; r1}{1 \; Bm}$ ...(1)

Put m_{AC} m in (1)

Ÿ m_1, m_2 tan D, cot D

a 10

' a^2 11a 3 $\left(m_1^2 \; \frac{1}{m_1^2}\right)$ 220

Ÿ 100 110 3 $\tan^2$ D $\cot^2$ D 220

Ÿ $\tan^2$ D 3 or $\frac{1}{3}$

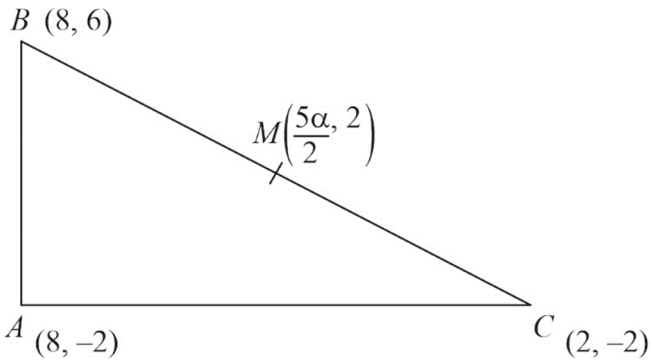
D $\frac{S}{3}, \frac{S}{6}$

Put a & D in given eqn

Ans. 128

11.(B) A D, 2 , B D6 , C $\left(\frac{D}{4}, 2\right)$

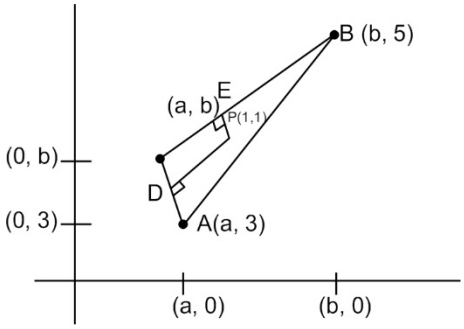
AC AAB



? $\frac{5 \; D}{8}$ 5

Ÿ D 8

12.(B)



m_{AC} • • o f

m_{PD} 0

D §a a b 3 / © 2 , 2

D §a, b 3 / © 2

m_{PD} 0

b 3 / 2 1 0

b 3 2 0

b 1

E §b a 5 b · ab§ / © 2 , 2 , 1 2©

m_{CB} . m_{EP} 1

§5 b · §2 1 · / ©b a , 1 a 1 , 1 / © 2 1 , 1

§ 6 · §2 · / ©1 a , 1 a© 3 , 1 1

12 1 a a 3

12 a^2 3a a 3

Ÿ a^2 2a 15 0

a 5 a 3 0

a 5 or a 3

Given ab ! 0

a 1 ! 0

a ! 0

a 0

a 3 Accept

AP line A 3,3 P 1,1

y 1 §3 1 · / ©3 1 , x 1 1

2y 2 x 1

Ÿ x 2y 3 Appling ...(1)

Line BC B 1,5

C 3, 1

y 5 6/2 x 1

y 5 3x 3

$y \quad 3x \quad 8 \qquad \ldots(2)$

Solving (1) and (2)

$x \quad 2\ 3x \quad 8 \quad 3$

$\ddot{Y} \qquad 7x \quad 16 \quad 3$

$7x \quad 13$

$x \quad \frac{13}{7}$

$y \quad 3\left(\frac{13}{7}\right) \quad 8$

$\frac{39 \quad 56}{7}$

$y \quad \frac{17}{7}$

$x \quad y \quad \frac{13 \quad 17}{7} \quad \frac{4}{7}$

13.(D) Point of intersection of given lines

$A\ 1,\ 2\ ,\ B\left(\frac{39}{1 \quad 2p},\frac{78}{1 \quad 2p}\right)$ and $C\left(\frac{3p \quad 39}{p \quad 1},\frac{36}{p \quad 1}\right)$

If $O\ 2,3$ is the circumcentre then

$OA^2 \quad OB^2 \quad OC^2 \quad \text{radius}^2$

$26 \quad \left(\frac{39}{1 \quad 2p} \quad 2\right)^2 \quad \left(\frac{78}{1 \quad 2p} \quad 3\right)^2 \quad \left(\frac{3p \quad 39}{p \quad 1} \quad 2\right)^2 \quad \left(\frac{36}{p \quad 1} \quad 3\right)^2$

$\ddot{Y}\ p \quad 8$

So the points are $A\ 1,\ 2\ ,B\left(\frac{13}{5},\frac{26}{5}\right)$ and $C \quad 7,4$

Area of triangle $ABC \quad \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 7 & \frac{13}{5} \\ 2 & 4 & \frac{26}{5} \end{vmatrix} \quad \frac{324}{10}$

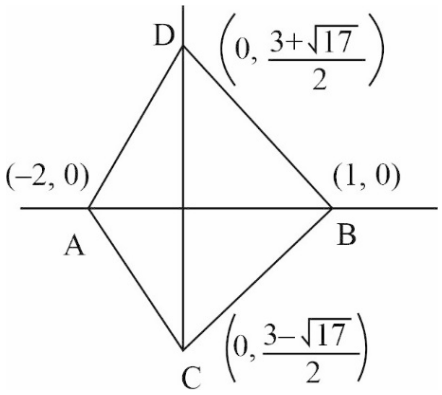
14.(B) $x \quad 1^2 \quad y \quad 2^2 \quad x \quad 2^2 \quad y \quad 1^2 \quad 14$

$x^2 \quad 2x \quad 1 \quad y^2 \quad 4 \quad 4y \quad x^2 \quad 4x \quad 4 \quad y^2 \quad 1 \quad 2y \quad 14$

$2x^2 \quad 2y^2 \quad 2x \quad 6y \quad 4 \quad 0$

$x^2 \quad y^2 \quad x \quad 3y \quad 2 \quad 0 \quad y \quad \frac{3 \quad r\sqrt{9 \quad 8}}{2} \quad \frac{3 \quad r\sqrt{n}}{2}$

y intercept $= y^2 \quad 3y \quad 2 \quad 0 \quad y \quad \frac{3 \quad r\sqrt{17}}{2}$



x intercept = $x^2 \quad x \quad 2 \quad 0 \quad x \quad 2, x \quad 1$

Area ABCD $\frac{1}{2}$ u3 u$\sqrt{17}$ $\frac{3}{2}\sqrt{17}$

15.(3) Coordinates of A 1, 2 , B $\left(\frac{15a}{1\ 2p}, \frac{30a}{1\ 2p}\right)$ and orthocentre H (2a,)

Slope of AH = p

a + 2 = p

a + 2 = p ...(1)

Slope of BH = – 1

31a – 2ab = 15a + 4p – 2 ...(2)

From (1) and (2)

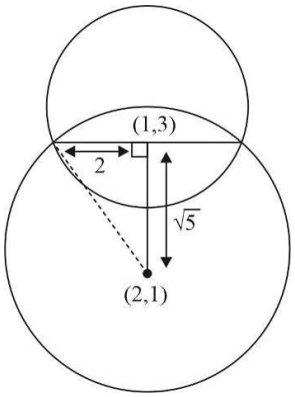
a 1 & p 3

Solutions ofArchive - JEE Main & Advanced

Circle	Class- XI | Mathematics

JEE Main 2021

1.(3)



Distance between (1,3) and (2,1) $\sqrt{5}$

$? \sqrt{5}^2 \quad (2)^2 \quad r^2$

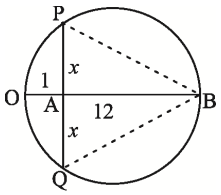
$r \quad 3$

2.(56.25) $(x \quad 5)^2 \quad y^2 \quad 9 \left(x \quad 5\right)^2 \quad y^2 \quad Ÿ \quad x^2 \quad y^2 \quad 10x \quad 25 \quad 9x^2 \quad 9y^2 \quad 90x \quad 225$

$Ÿ \quad 8x^2 \quad 8y^2 \quad 100x \quad 200 \quad 0 \quad Ÿ \quad x^2 \quad y^2 \quad \frac{25}{2}x \quad 25 \quad 0$

$r \quad \sqrt{\left(\frac{25}{4}\right)^2 \quad 25} \quad Ÿ \quad r^2 \quad \frac{625}{16} \quad 25 \quad Ÿ \quad 4r^2 \quad \frac{625}{4} \quad 100 \quad \frac{225}{4} \quad 56.25$

3.(A)

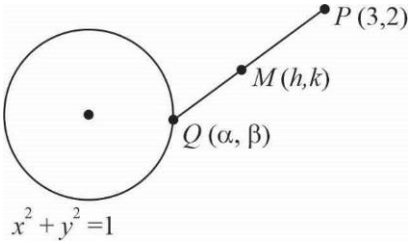


Power of the point about A

$1.12 \quad x \quad x \quad Ÿ \quad x \quad \sqrt{12}$

Area of $'PQB \quad \frac{1}{2}PQ \quad AB \quad \frac{1}{2} \quad 2x \quad 12 \quad 12\sqrt{12} \quad 24\sqrt{3}$

4.(D)



Let Q D, E be a variable point lying on the circle. M is the mid point of P & Q.

Ÿ h $\frac{D\ 3}{2}$ & k $\frac{E\ 2}{2}$ Ÿ D 2h 3& E 2k 2

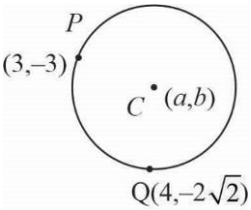
Now; Q D, E satisfies x^2 y^2 1

Ÿ D² Ê 1 Ÿ2h 3 ² 2k 2 ² 1 Ÿ 4h² 4k² 12h 8k 12 (

Ÿ h^2 k^2 3h 2k 3 0 ? Locus is x^2 y^2 3x 2y 3 0 (circle)

Radius r $\sqrt{\frac{9}{4}\ 1\ 3}$ $\frac{1}{2}$

5.(9) To satisfy the given condition, the curve must be a circle with centre C a, b & circle passes through P 3, 3 & Q 4, $2\sqrt{2}$



CP² CQ² Ÿ a 3 ² b 3 ² a 4 ² b $2\sqrt{2}$ ²

Ÿ a 3 $2\sqrt{2}$ b 3 .....(i)

Also, a $2\sqrt{2}$ b 3 (given) .....(ii)

(i) & (ii) Ÿ a 3,b 0 Ÿ a^2 b^2 ab 9

6.(1225) Normal at (5, 7) is y 3 $\frac{7\ 3}{5\ 2}$(x 2)

Ÿ y 3 $\frac{4}{3}$(x 2) Ÿ 3y 9 4x 8

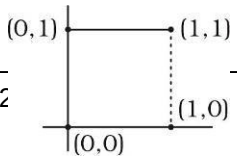
Ÿ 3y 9 4x 8 Ÿ 4x 3y 1 0

Equation of tangent is 3x 4y O 0 (5, 7) lies on it

Ÿ 15 28 O 0 Ÿ O 43 Ÿ 3x 4y 43 0

Area $\frac{1}{2}$ u $\frac{43}{3}$ $\frac{1}{4}$ u 7 ? 24 A $\frac{1}{2}$ u $\frac{175}{12}$ u 24 1225

7.(A) Centre of the circles are (0, 0)



(1, 0)

(1, 1)

(0, 1)

Which are the sides of a square

8.(A) The circles have their centre as radii

$C_1 \{(5,5)$; r_1 3

$C_2 \{(11,5)$; r_2 3

C_1C_2 r_1 r_2

The circles touch externally.

9.(A) $2\sqrt{g^2 \quad c} \quad 2\sqrt{\frac{a^2}{4} \quad c} \quad 2\sqrt{2} \qquad \ddot{Y} \qquad \frac{a^2}{4} \quad c \quad 2 \qquad \ldots$ (i)

$2\sqrt{f^2 \quad c} \quad 2\sqrt{5} \quad \ddot{Y} a^2 \quad c \quad 5 \qquad \ldots$ (ii)

$a^2 \quad 4 \quad \ddot{Y} a \quad 2$ and $c \quad 1$

Equation of circle $x^2 \quad y^2 \quad 2x \quad 4y \quad 1 \quad 0$

$x \quad 1^2 \quad y \quad 2^2 \quad 6$ ……(ii)

Equation of tangent

$y \quad 2 \quad m \; x \quad 1 \quad r\sqrt{6}\sqrt{1 \quad m^2}$

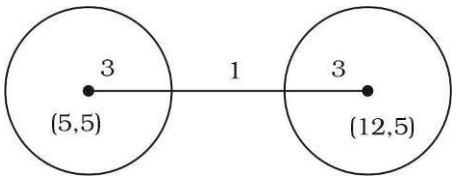
Put $m \quad 2$ as the tangent is perpendicular to $x \quad 2y \quad 0$

$y \quad 2 \quad 2x \quad 2 \quad r\sqrt{6}\sqrt{5}$

$2x \quad y \quad r\sqrt{30} \quad 0$

Distance from origin $\left|\frac{0 \quad 0 \quad r\sqrt{30}}{\sqrt{5}}\right| \quad \sqrt{6}$

10.(1)



mindist. between two circle is1

11.(B) Eqn. of circle should be $x \quad 2^2 \quad y \quad 5^2 \quad O2x \quad y \quad 1 \quad 0$

Centre $\left(\frac{2 \; O \; 4}{2}, \frac{O10}{2}\right)$ $\left(\frac{O \; 10}{2}\right)$ must lie on $x \quad 2y \quad 2 \quad O \quad O10 \quad 4 \quad 2\ddot{Y} \quad O12$

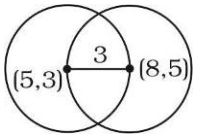
$O \quad 6$

Centre $8,2 \qquad r \quad \sqrt{36 \quad 9} \quad 3\sqrt{5}$

12 .(C) C_1 5,5 r_1 3

C_2 8,5 r_2 3

C_1C_2 3



13 .(D) Let the centre of variable circle be C(h ,k) and radius = r

$\sqrt{(h \quad 0)^2 \quad (k \quad 0)^2} \quad 3 \quad r$

Ÿ $h^2 \quad k^2 \quad (3 \quad r)^2$ …(i)

& $\sqrt{(h \quad 2)^2 \quad (k \quad 0)^2} \quad r \quad 1$

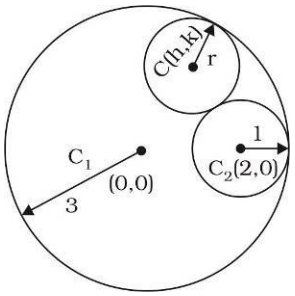
Ÿ $(h \quad 2)^2 \quad k^2 \quad (r \quad 1)^2$ …(ii)

Ÿ $h^2 \quad 4h \quad 4 \quad k^2 \quad (r \quad 1)^2$

From (i) $(3 \quad r)^2 \quad 4h \quad 4 \quad (r \quad 1)^2$

$(3 \quad r)^2 \quad (r \quad 1)^2 \quad 4 \quad 4h$

Ÿ $9 \quad r^2 \quad 6r \quad r^2 \quad 2r \quad 1 \quad 4 \quad 4h$ Ÿ $12 \quad 8r \quad 4h$ Ÿ $h \quad 3 \quad 2r$ Ÿ $r \quad \frac{3 \quad h}{2}$



From (i) $h^2 \quad k^2 \quad (3 \quad r)^2$ $\left(3 \quad \frac{(3 \quad h)}{2}\right)^2 \cdot \left(\frac{3 \quad h}{2}\right)^2$

Ÿ $4h^2 \quad 4k^2 \quad 9 \quad h^2 \quad 6h$ Ÿ $3h^2 \quad 6h \quad 4k^2 \quad 9 \quad 0$

Locus

$3x^2 \quad 6x \quad 4y^2 \quad 9 \quad 0$

Passes through the points 2, r3 / 2

14 .(1) $C_1C_2 \quad |1 \quad r|$

$\sqrt{2}r \quad |1 \quad r|$ Ÿ $2 \quad r^2 \quad (1 \quad r)^2$ Ÿ $r \quad \sqrt{2} \quad 1$

Equation of CE : $y \quad 1 \quad m \quad x \quad 1$

Ÿ $mx \quad y \quad 1 \quad m \quad 0$

It is tangent to the smaller circle

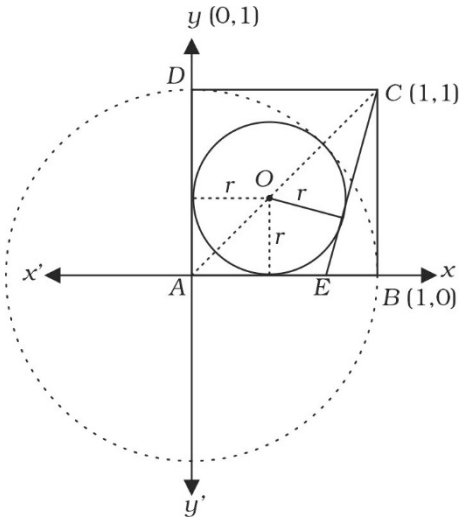
? $\left|\frac{mr \quad r \quad 1 \quad m}{\sqrt{1 \quad m^2}}\right| \quad r$

Ÿ $\frac{r \quad 1^2 \quad m \quad 1^2}{1 \quad m^2} \quad r^2$

Ÿ $\frac{m \quad 1^2}{1 \quad m^2} \quad \left(\frac{r}{1}\right)^2$, put $r \quad \sqrt{2} \quad 1$

On solving we get $m \quad 2 \quad r\sqrt{3}$

Taking greater slope $m \quad 2 \quad \sqrt{3}$



? Equation of CE : $y - 1 = -2\sqrt{3}(x - 1)$

Put $y = 0 \Rightarrow x - 1 = \frac{1}{2\sqrt{3}} \Rightarrow 1 - x = \frac{1}{2\sqrt{3}}$

? BE $1 - x = \frac{1}{2\sqrt{3}} \cdot \frac{2\sqrt{3}}{2\sqrt{3}} = 2\sqrt{3} \Rightarrow D = -2, E = -1$? $D = E = 1$

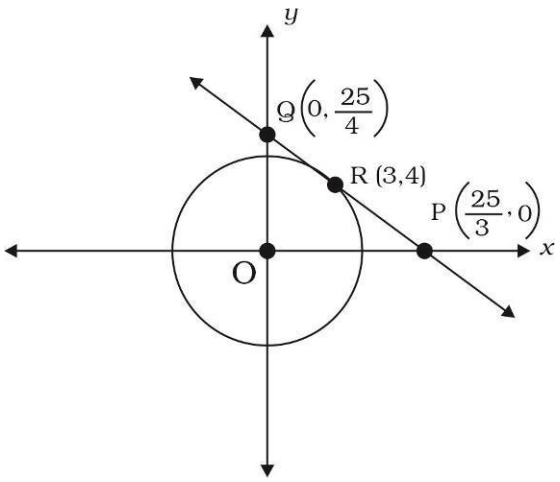
15.(B) Equation of tangent to the circle $x^2 + y^2 = 25$ at R(3,4) will be $xx_1 + yy_1 = 25$

$3x + 4y = 25$

$\Rightarrow P\left(\frac{25}{3}, 0\right)$ & $Q\left(0, \frac{25}{4}\right)$

$OP = \frac{25}{3}$, $OQ = \frac{29}{4}$; $PQ = \frac{125}{12}$

$\Rightarrow$ Incentre $(I) \equiv \left(\frac{25}{12}, \frac{25}{12}\right)$



This incentre will act as the centre of the required circle passing through origin.

Hence, radius of required circle

$(r) = \sqrt{\left(\frac{25}{12}\right)^2 + \left(\frac{25}{12}\right)^2} = \sqrt{\frac{625}{72}} \Rightarrow r^2 = \frac{625}{72}$

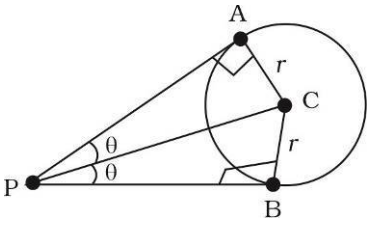
16.(D) $\angle APB = \tan^{-1}\left(\frac{12}{5}\right)$

$\Rightarrow 2\theta = \tan^{-1}\left(\frac{12}{5}\right)$ $\Rightarrow \tan 2\theta = \frac{12}{5} \Rightarrow \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{12}{5}$

$\Rightarrow 6\tan^2\theta + 5\tan\theta - 6 = 0 \Rightarrow \tan\theta = \frac{3}{2}, -\frac{2}{3}$

Since, $2\theta \in (0, \pi)$ (given) $\Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$

Hence, $\tan\theta = \frac{2}{3}$ $\tan\theta = -\frac{3}{2}$ rejected



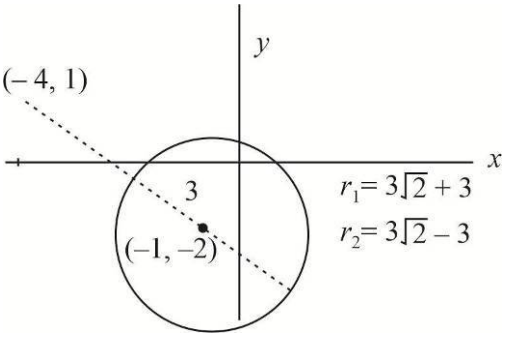
In $\triangle PAC$; $\angle A = 90°$ $\Rightarrow \frac{AC}{AP} = \tan\theta = \frac{2}{3} \Rightarrow AP = \frac{3r}{2}$

Now, $\frac{\text{ar. } \triangle APB}{\text{ar. } \triangle CAB} = \frac{\frac{1}{2}(AP)(PB)\sin 2\theta}{\frac{1}{2}(AC)(BC)\sin(\angle ACB)} = \frac{\frac{1}{2}\left(\frac{3r}{2}\right)\left(\frac{12}{13}\right)}{\frac{1}{2}(r)(r)\left(\frac{12}{13}\right)} = \frac{9}{4}$

$\because \tan 2\theta = \frac{12}{5} \Rightarrow \sin 2\theta = \frac{12}{13}$ & $\angle ACB = 180° - \angle APB$

because ACBP are concyclic points

17.(B) $x^2 + y^2 + 2x - 4y - 4 = 0$ centre $(-1, 2)$ $r = \sqrt{1 + 4 + 4} = 3$



$\frac{r_1}{r_2}$ $\frac{\sqrt{2}\ 1}{\sqrt{2}\ 1}$ $\frac{2\ 1\ 2\sqrt{2}}{1}$ 3 $2\sqrt{2}$ Ÿ a b 5

18.(C) $36x^2$ $36y^2$ 108 x 120 y C 0

Acc. to question; x -intercept 0 & y intercept 0 { ' circle is neither touches nor cuts axes}

Ÿ g^2 c 0 & f^2 c 0

Ÿ $\left(\frac{54}{36}\right)^2 - \frac{C}{36}$ 0 & $\left(\frac{60}{36}\right)^2 - \frac{C}{36}$ 0 Ÿ $\frac{9}{4} - \frac{C}{36}$ 0 & $\frac{25}{9} - \frac{C}{36}$ 0

Ÿ C ! 81 & C ! 100 Ÿ C ! 100 ….(i)

Now; Point of intersection of x 2y 4 & 2x y 5 i.e. P 2, 1 lies inside circle

Ÿ S_1 0 Ÿ $36\ 2^2$ $36\ 1^2$ 108 2 120 1 C 0

Ÿ C 156 ….(ii)

(i) & (ii) Ÿ 100 C 156

19.(1) If $\bar{z}$ x iy $\bar{z}$ x^2 y^2 2xy

Re($\bar{z}^2$) x^2 y^2

Im($\bar{z}$) y ; Re($\bar{z}$) x

x^2 y^2 $2y^2$ 2x 0 Ÿ x^2 y^2 2x 0

Centre { (1, 0)

The parabola is : x^2 6x 13 y

$(x\ 3)^2$ y 4

Vertex (3, 4)

Line passing through (–1, 0) and (3, 4)

y x 1 , y intercept is 1.

20.(A) Circle A and Circle B

Should lie completely inside Circle C

$A \{ x^2 \quad y^2 \quad x \quad y \quad \frac{1}{2} \{ \left(x \quad \frac{1}{2}\right)^2 \quad \left(y \quad \frac{1}{2}\right)^2 \quad 1$

$B \{ x^2 \quad y^2 \quad 4y \quad \frac{7}{4} \quad 0 \{ (x \quad 0)^2 \quad (y \quad 2)^2 \quad \left(\frac{3}{2}\right)^2$

$C \{ (x \quad 2)^2 \quad (y \quad 1)^2 \quad dr^2$

A lies inside C

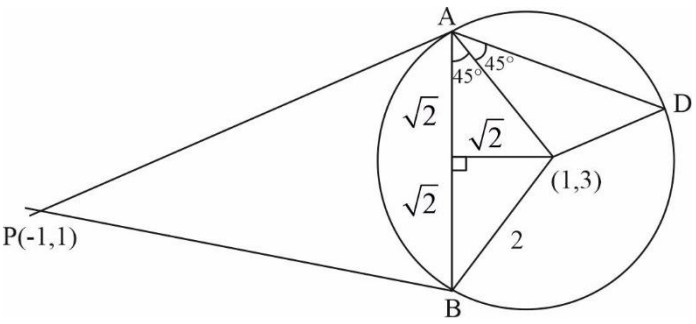
$|r \quad 1| t \sqrt{\left(2 \quad \frac{1}{2}\right)^2 \quad \left(1 \quad \frac{1}{2}\right)^2} \quad \sqrt{\frac{9}{4} \quad \frac{1}{4}} \quad \sqrt{\frac{5}{2}}$

B lies inside C

$\left|r \quad \frac{3}{2}\right| t \sqrt{4 \quad 1} \quad \sqrt{5}$

$|r|_{min} \quad \frac{3}{2} \quad \sqrt{5}$

21.(B) $x^2 \quad y^2 \quad 2x \quad 6y \quad 6 \quad 0$



AB $x \quad y \quad (x \quad 1) \quad 3(y \quad 1) \quad 6 \quad 0$

$x \quad y \quad x \quad 1 \quad 3y \quad 3 \quad 6 \quad 0 \quad 2x \quad 2y \quad 4 \quad 0 \Ÿ x \quad y \quad 2$

AB $2\sqrt{2}$

Area $('ABD) \quad \frac{1}{2} u 2\sqrt{2} u 2\sqrt{2} \quad 4$

22.(A) $'OCP \quad 90q$

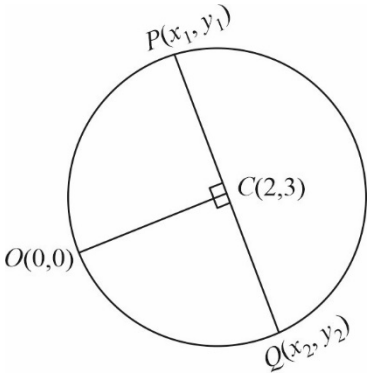
$\frac{y \quad 3}{x \quad 2} u \frac{3 \quad 0}{2 \quad 0} \quad 1 \Ÿ y \quad 3 \quad \frac{2}{3}(x \quad 2)$

$(x \quad 2)^2 \quad (y \quad 3)^2 \quad (2 \quad 0)^2 \quad (3 \quad 0)^2$

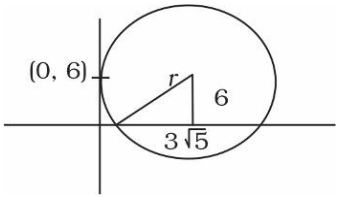
$\Ÿ (x \quad 2)^2 \quad \frac{4}{9}(x \quad 2)^2 \quad 13$

$\Ÿ \frac{13}{9}(x \quad 2)^2 \quad 13 \Ÿ (x \quad 2)^2 \quad 9 \Ÿ x \quad 2 \quad 3, 2 \quad 3$

$x \quad 5, \quad 1; \quad \{P, Q\} \quad \{(\quad 1,5), (5,1)\}$



23.(D) $4\sqrt{36-45}$



$r = \sqrt{81}$

$r = 9$

24.(61) $r = \sqrt{\frac{p^2}{4} + \frac{1+p^2}{4} + 5} = \frac{\sqrt{2p^2+2p+19}}{2}$ $r = 0.5\sqrt{2p^2+2p+19} \geq 100$

$p \in \left(-\infty, \frac{1-\sqrt{239}}{2}\right] \cup \left[\frac{-1+\sqrt{239}}{2}, \infty\right)$

? Number of integral value of p^2 is 61.

25.(B) $C: (x-2)^2 + (y-1)^2 = 0(x+2y)$

$C_1: x^2 + y^2 + 2y - 5 = 0$

Centre of C_1 {(0, –1)

The common chord of C and C_1 is: $2(2x-2) + 1(2y+1) = 0(x+2y) \Rightarrow 2y - 5 = 0$

and it passes through: (0, –1)

$-3 = 0(2) \Rightarrow 2 - 5 = 0 \Rightarrow \lambda = 7$

$C: (x-2)^2 + (y-1)^2 = 7(x+2y) = 0$

$x^2 + y^2 - 11x - 12y + 5 = 0$

$D = 7\sqrt{5}$

26.(16) Let the given points be $A(0,0), B(1,0), C(0,1)$ and $D(1,1)$.

Let $P(x,y)$ be the point whose locus is to find.

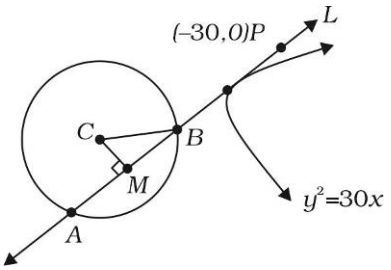
$PA^2 + PB^2 + PC^2 + PD^2 = 18 \Rightarrow (x^2+y^2) + ((x-1)^2+y^2) + (x^2+(y-1)^2) + ((x-1)^2+(y-1)^2) = 18$

$\Rightarrow 4x^2 + 4y^2 - 4x - 4y - 14 = 0$

$\Rightarrow x^2 + y^2 - x - y - \frac{7}{2} = 0$ is a circle of radius $\sqrt{\frac{1}{4}+\frac{1}{4}+\frac{7}{2}} = \sqrt{4} = 2$

$\Rightarrow$ Diameter = 4 = d $\Rightarrow d^2 = 16$

27.(C) Let L be the tangent to the parabola y^2 30x.



? L : y mx $\frac{a}{m}$ mx $\frac{15}{2m}$

It passes through P 30,0

? 0 30m $\frac{15}{2m}$ Ÿ m r$\frac{1}{2}$? L : y r$\frac{1}{2}$x r$\frac{15}{2 \cdot \frac{1}{2}}$,

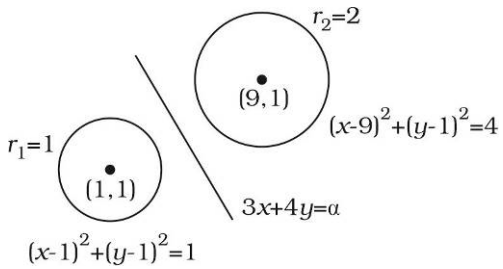
i.e. y $\frac{1}{2}$x 15; y $\frac{1}{2}$x 15

i.e. x 2y 30 0; x 2y 30 0

Given circle has centre and radius as 15,0 and $\frac{15}{2}$ respectively.

AB 2MB $2\sqrt{BC^2 \quad CM^2}$ $2\sqrt{\left(\frac{15}{2}\right)^2 \cdot \left(\frac{|15 \quad 30|}{\sqrt{1 \quad 4}}\right)^2}$ $3\sqrt{5}$

28 .(165)



Both centres should lie on either side of the line as well as line can be tangent to circle.

(3 4 D) (27 4)D 0

(7 D) (31)D 0 Ÿ D(7,31) …(i)

d_1 distance of (1,1) from line

d_2 distance of (9,1) from line

d_1 tr_1 Ÿ $\frac{|7 \quad D}{5}$ t 1 Ÿ D• (f ,2] %[12, f) …(ii)

d_2 tr_2 Ÿ $\frac{|31 \quad D}{5}$ t 2 Ÿ D• (f ,21] %[41, f) …(iii)

(i) ^ (ii) ^ (iii) Ÿ D• [12,21]

Sum of integers = 165

29 .(40) Circle touches at 1,2 and common tangent 4x 3y 10

Centre lies at 3x 4y 5 0

tan T $\frac{3}{4}$ Ÿsin T $\frac{3}{5}$ r, cos T $\frac{4}{5}$ r (same sign)

Centre of both circles of radius 5 lie at 3x 4y 5 0 and touches at 1,2

centres 1 r cos T, 2 r sin T 1 5 § 4 r, 2 5 § 3 r 1 r 4, 2 r 3 5,5 or 3, 1

Now | D E J | G | 10 4u | 40

30 .(B) x^2 y^2 d4 ……(i)

No. of integral points satisfying 0,0 , 1,0 , 2,0 , 1,0 , 2,0 , 1,1 , 1, 1 , 1,1 , 1, 1 ,

0, 1 , 0, 1 , 0, 2 , 0, 2

x 2 ^{2} y^2 d4

No. of integral points satisfying x 2 ^{2} y^2 d4

No. of integral points satisfying (2, 0), 3,0 , 4,0 , 1,0 , 0,0 , 3,1 , 3, 1 , 1,1 , 1, 1 ,

2, 1 , 2, 1 , 2, 2 , 2, 2 ,

x 2 ^{2} y 2 ^{2} d4

No. of integral points satisfying (2, 2), 3,2 , 4,2 , 1,2 , 0,2 , 3,3 , 3,1 , 1,3 , 1,1 ,

2, 0 , 2, 1 , 2, 3 , 2, 4

Common points Ÿ 0,0 , 1,0 , 2,0 1,1 , 1, 1 of C_1 and C_2

Similarly points common in C_2 and C_3 Ÿ 5 points : (1, 1), (2, 0), (2, 1), (2, 2), (3, 1)

No. of relations $2^{5 \text{ u}5}$ 2^{25}

JEE Advanced 2021

1.(D) C_n lies inside M iff distance between centres $r \quad a_n$

$S_{n\ 1} \quad r \quad a_n$ Ÿ $r \ !S_{n\ 1} \quad a_n$ Ÿ $r \ !S_n$

$S_n \quad \frac{1 \quad 1/2^{\ n}}{1/2} \quad 2 \quad 2^{1\ n}$; $r \ !S_n$ iff $\frac{1025}{513} \ !2 \quad 2^{1\ n}$

iff $2^{1\ n} \ !\frac{1}{513}$

True for $n \ d10$ Ÿ $k \quad 10$

Consider C_{n_1} and C_{n_2} $\{n_1 \ !n_2\}$

C_{n_1} and C_{n_2} will not intersect iff distance between their centres exceeds sum of their radii

Ÿ $S_{n_1\ 1} \quad S_{n_2\ 1} \ !a_{n_1} \quad a_{n_2}$ Ÿ $S_{n_1\ 1} \quad a_{n_1} \ !S_{n_2}$

Ÿ $2 \quad 2^{1\ (n_2\ 1)} \quad \frac{1}{2^{n_1\ 1}} \ !2 \quad 2^{1\ n_2}$ Ÿ $2^{n_1\ n_2} \ !3$ Ÿ $n_1 \quad n_2 \ t2$

We can select at most 5 circles from the $k \quad 10$ circles lying inside M such that no pair of circl es intersect

Ÿ A 5

2.(B) D_n lies inside M iff $\sqrt{2}S_{n\ 1}$ distance between the 2 centres $r \quad a_n$

$r \ !\sqrt{2}S_{n\ 1} \quad a_n$

$2\sqrt{2} \quad \frac{\sqrt{2}}{2^{198}} \ !\sqrt{2}(2 \quad 2^{1\ (n\ 1)}) \quad 2^{1\ n}$

$2\sqrt{2} \quad \frac{\sqrt{2}}{2^{198}} \ !2\sqrt{2} \quad 2\sqrt{2} \quad 2^{1\ n} \quad 2^{1\ n}$

$2^{1\ n}(2\sqrt{2} \quad 1) \quad \frac{\sqrt{2}}{2^{198}}$ Ÿ $2^n \quad 2^{199}$ §:2 © $\frac{1}{\sqrt{2}}$:

True for $n \ d199$

Solutions ofArchive - JEE Main & Advanced

Circle	Class- XI | Mathematics

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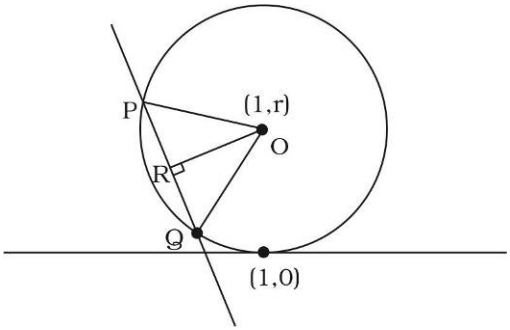
1.(7) $(x-h)^2+(y-k)^2=r^2$ touches x-axis

At (1,0)

$\Rightarrow (1-h)^2+k^2=r^2 \Rightarrow h^2+k^2-2h+1=r^2$

$\Rightarrow h=1$ & $k=r$

Distance from $(1, k)$ on $x+y=0$ is $d=\left|\frac{1+k}{\sqrt{2}}\right|=\left|\frac{1+r}{\sqrt{2}}\right|$



$\Rightarrow r^2=1+\frac{(1+r)^2}{2}$

$\Rightarrow 2(r^2-1)=r^2+1+2r$

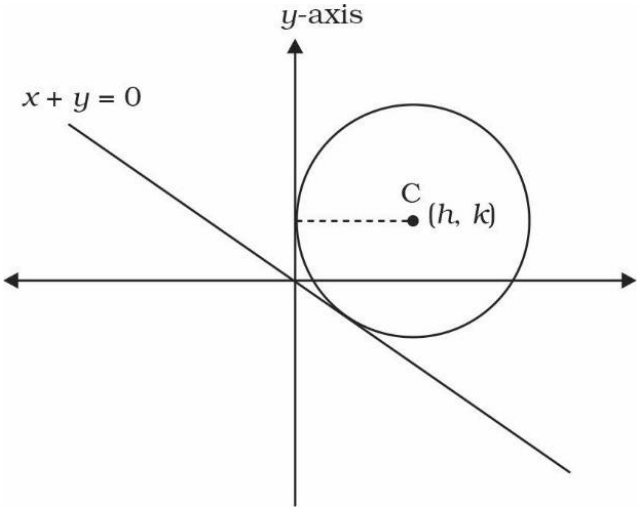
$r^2-2r-3=0$

$\Rightarrow r=3$ or $r=-1$ (rejected)

$\Rightarrow k=3$

$\Rightarrow h+k+r=1+3+3=7$

2.(D)



Perpendicular Drawn from $(-h, k)$

On x y 0 is

$\left|\frac{h \quad k}{\sqrt{2}}\right| \quad |h|$

Squaring both side

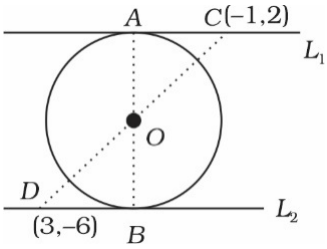
Ÿ $h^2 \quad k^2 \quad 2hk \quad 2h^2$

Ÿ $h^2 \quad k^2 \quad 2hk \quad 0$

Ÿ $x^2 \quad y^2 \quad 2xy \quad 0$

Ÿ $x^2 \quad y^2 \quad 2xy$

3.(C)



1,2 lies on L_1

So 4 u 1 3 2 k_1 0

k_1 10 ; 3, 6 lies on L_2

4 u3 3 u 6 k_2 0 ; 12 18 k_2 0 Ÿk_2 30

$|AB| \quad \frac{|10 \quad 30|}{\sqrt{16 \quad 9}} \quad \frac{40}{5} \quad 8$

So radius = 4

Center of circle = midpoint of CD $\left(\frac{3 \quad 1}{2}, \frac{2 \quad 6}{2}\right)$; 1, 2

So equation of circle Ÿ x 1^2 y 2^2 4^2

4.(C) f x y 2f x f y

Given f 1 2

f 2 f 1 1 2 f 1 . f 1 2.2^2 2^3

f 3 f 2 1 2 f 2 f 1 2 2 2^3 2u 2^5

$\left.\begin{matrix}10\\ \\ k\ 1\end{matrix}\right|$ f D k $\left.\begin{matrix}10\\ \\ k\ 1\end{matrix}\right|$ f D k 2f $\left.\begin{matrix}10\\ \\ k\ 1\end{matrix}\right|$ D f x

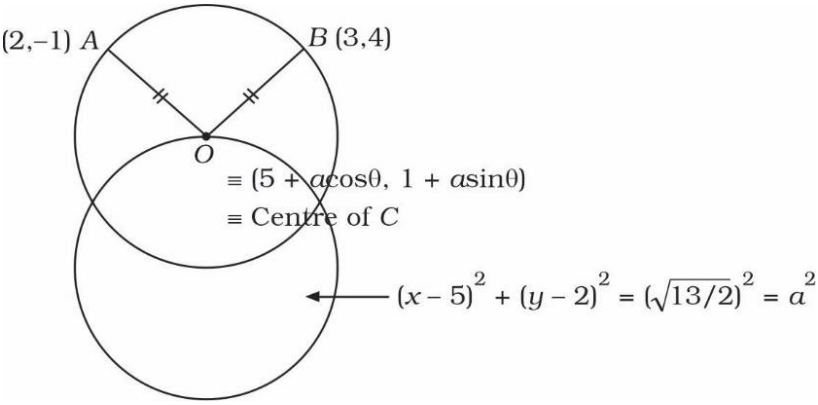
2 f D f 1 f 2 f 3 ..... f 10

$2f = 2 + 2^3 + 2^5 + \ldots\ldots\ldots$, $2f = \dfrac{2\left(2^{20}-1\right)}{4-1}$,

$2f = \dfrac{2\left(2^{20}-1\right)}{3} - \dfrac{512}{3} = 2^{20} - 1$; $4f = 512 \Rightarrow f = \dfrac{512}{4} = 128$

$f = 128 \Rightarrow d = 4$

5.(B)



OA = OB

$\Rightarrow (5 - a\cos\theta_2)^2 + (1 - a\sin\theta_1)^2 = (5 - a\cos\theta_3)^2 + (1 - a\sin\theta_4)^2$

$\Rightarrow 9 - 6a\cos\theta + a^2\cos^2\theta + 4 - 4a\sin\theta + a^2\sin^2\theta$

$= 4 - 4a\cos\theta + a^2\cos^2\theta + 9 - 6a\sin\theta + a^2\sin^2\theta$

$100\sin\theta = 20\cos\theta$

$\Rightarrow \tan\theta = \dfrac{1}{5}$ $\begin{cases} \sin\theta = 1/\sqrt{26} \\ \cos\theta = -5/\sqrt{26} \end{cases}$ $\begin{cases} \sin\theta = -1/\sqrt{26} \\ \cos\theta = 5/\sqrt{26} \end{cases}$

Hence centre of circle C may be $\left(\dfrac{\sqrt{13}}{\sqrt{2}}\cdot\dfrac{5}{\sqrt{26}}+1, \dfrac{\sqrt{13}}{\sqrt{2}}\cdot\dfrac{1}{\sqrt{26}}+2\right) = \left(\dfrac{3}{2}, \dfrac{5}{2}\right)$ or $\left(\dfrac{5}{2}, \dfrac{1}{2}\right)$

If radius of the centre C is are than $r^2 = \left(2 - \dfrac{3}{2}\right)^2 + \left(1 - \dfrac{5}{2}\right)^2$ or $\left(2 - \dfrac{5}{2}\right)^2 + \left(1 - \dfrac{1}{2}\right)^2$

$\dfrac{1}{4} + \dfrac{25}{4}$ or $\dfrac{121}{4} + \dfrac{9}{4}$

$\dfrac{26}{14}$ or $\dfrac{130}{4}$

$r^2 = \dfrac{13}{2}$ or $\dfrac{65}{2}$

6.(D) $C : 4x^2 + 4y^2 - 12x + 8y + k = 0$

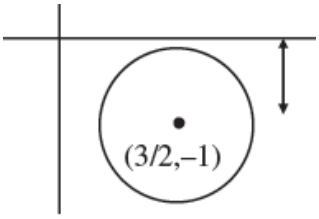
$\Rightarrow x^2 + y^2 - 3x + 2y + \frac{k}{4} = 0$

Centre $\left(\frac{3}{2}, -1\right)$; $r = \sqrt{\frac{13 - k}{2}} > 0 \Rightarrow k < 13$ ...(1)

(i) Point $\left(1, \frac{1}{3}\right)$ lies on or inside circle C

$\Rightarrow S_1 \geq 0 \Rightarrow k \geq \frac{92}{9}$ ...(2)

(ii) C lies in 4 th quadrant



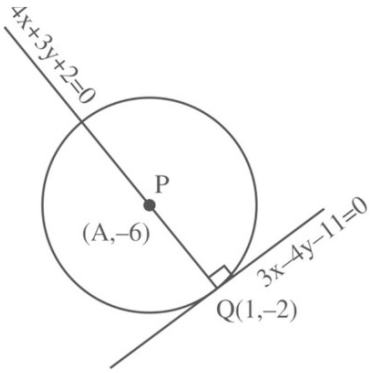
$r \leq 1$

$\Rightarrow \frac{\sqrt{13 - k}}{2} \leq 1$

$\Rightarrow k \geq 9$ ....(3)

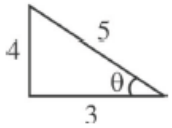
Hence (1) $\cap$ (2) $\cap$ (3) $\Rightarrow k \in \left[9, \frac{92}{9}\right)$

7.(11)



$4x + 3y + 2 = 0$

$3x - 4y - 11 = 0$



$\frac{x}{25} \quad \frac{y}{50} \quad \frac{1}{25}$

$\frac{x \quad 1}{\cos T} \quad \frac{y \quad 2}{\sin T}$ r5

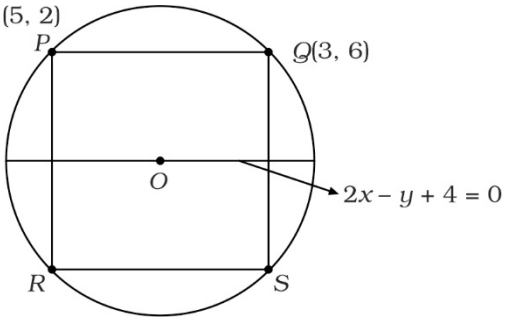
y 2 5 $\left(\frac{4}{5}, 6\right)$

x 1 5 $\left(\frac{3}{5}, 4\right)$

Req. distance

$\left|\frac{5(4) \quad 12(\ 6) \quad 51}{13}\right| \quad \left|\frac{20 \quad 72 \quad 51}{13}\right| \quad \frac{143}{13} \quad 11$

8.(16)



PQ||2 x y 4 0

PQ $2\sqrt{5}$

Pt R will be image of Pt 'P' in 2x y 4 0

So, Let R D, E

$\frac{E \ 2}{D \ 1} \quad \frac{1}{2}$ Ÿ 2 E 4 D1

D 2 E 5 …(1)

& Mid Pt of 'PR' will be on 2x y 4 0

$2\left(\frac{D \ 1}{2}, \frac{E \ 2}{2}\right) \ 4 \quad 0$

2 D 5 E2

2 D E 8 …(2)

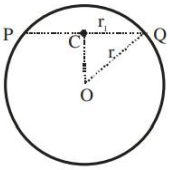
From (1) & (2)

D $\frac{11}{5}$, E $\frac{18}{5}$

PR $\frac{8}{5}\sqrt{5}$

Area of Rectangle PQ uPR AB $2\sqrt{5} \ u\frac{8\sqrt{5}}{5}$ 16

9.(10) PQ is diameter of circle



$S : x^2 \quad y^2 \quad 2\sqrt{2}x \quad 6\sqrt{2}y \quad 14 \quad 0$

$C \quad \sqrt{2}, 3\sqrt{2}$,0 2 $\sqrt{2}$,2 $\sqrt{2}$

$r_1 \quad \sqrt{6}$

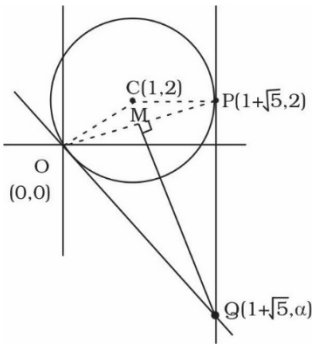
$S_1 : \quad x \quad 2\sqrt{2}^{\,2} \quad y \quad 2\sqrt{2}^{\,2} \quad r^2$

Now is 'OCQ

$|OC|^2 \quad |CQ|^2 | \quad OQ4^2 \quad 6 \quad r^2$

$r^2 \quad 10$

10.(C)



Slope of CO = 2

Slope of OQ $\quad \frac{1}{2}$

$\frac{D \quad 0}{1 \quad \sqrt{5} \quad 0} \quad \frac{1}{2}$

$D \quad \frac{1}{2} 1 \quad \sqrt{5}$

Equation of OP is $y \quad \frac{2}{1 \quad \sqrt{5}} x$

Area of OPQ $\quad \frac{1}{2} OP \ uQM$

Area of OPQ. $\quad \frac{1}{2} u\sqrt{4 \quad 1 \quad \sqrt{5}^{\,2}} \quad u \left| \frac{\frac{1}{2} 1 \quad \sqrt{5} \quad \frac{2}{1 \quad \sqrt{5}} . 1 \quad \sqrt{5}}{\sqrt{1 \quad \frac{4}{1 \quad \sqrt{5}^{\,2}}}} \right|$

On simplification we get $\quad \frac{5 \quad 3\sqrt{5}}{2}$

11.(No options match)

Since ‘ BAC $\frac{S}{2}$,BC is the diameter

Ÿ BC 2

AB $\sqrt{2}$ Ÿ AC $\sqrt{2}$

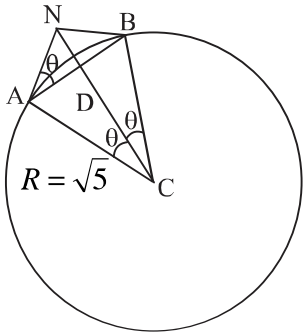
Ÿ Ar (’ABC) $\frac{1}{2}$ u$\sqrt{2}$ u$\sqrt{2}$ 1

12.(C) Equation of AB y x 2

C: Centre of C $_2$: (3, 2)

Length of perpendicular from (3, 2) to AB $\frac{3}{\sqrt{2}}$

Radius of C_2 $\sqrt{5}$



In the above figure cos T $\frac{CD}{AC}$ $\frac{\frac{3}{\sqrt{2}}}{\sqrt{5}}$ $\frac{3}{\sqrt{10}}$

Ÿ sin T $\frac{1}{\sqrt{10}}$

Area (’ANB) AD uND

(AD)^2 tan T

AD R sin T

Ÿ Area $\frac{R^2 \sin^3 T}{\cos T}$ $\frac{5}{3}$ u$\frac{1}{10}$ $\frac{1}{6}$

13.(25) S_1 : x^2 y^2 6x 8y 16 0 r$_1$ 3

S_2 : x^2 y^2 2(3 $\sqrt{3}$)x 2(4 $\sqrt{6}$)y k 6$\sqrt{3}$ 8$\sqrt{6}$

S_1 S_2 0 is common tangent

2x ~$\sqrt{3}$ 2$\sqrt{6}$y 16 k 6$\sqrt{3}$ 8$\sqrt{6}$ 0

2$\sqrt{3}$x 2$\sqrt{6}$y 16 k 6$\sqrt{3}$ 8$\sqrt{6}$ 0

$\left|\frac{k \quad 16}{\sqrt{12 \quad 24}}\right|$ 3 k 16 18

k 2

Centre of S_1 (3, 4) radius = 3

Centre of S_2 $\sqrt{3}$ 3, $\sqrt{6}$, 4 radius = 6

Hence $\frac{D\ \sqrt{3}\ \ 3}{2}$ 3

$D\ \sqrt{3}\ \ 3$

$\frac{E\ \sqrt{6}\ \ 4}{2}$ 4

$E\ \sqrt{6}\ \ 4$

$D\ \sqrt{3}^{\,2}$ $E\sqrt{6}^{\,2}$ $3^2\ \ 4^2\ \ 25$

14.(D) $x^2\ \ y^2\ \ 2gx\ \ 6y\ \ 19c\ \ 0$

Passing the (6, 1)

$36 + 1\ -12g + 6\ -19c = 0$

$12g + 19c = 43$ … (1)

Centre lies on $x - 2cy = 8$

Centre $(g, -3)$

$g\ \ 6c\ \ 8$ … (2)

From (1) and (2)

$12(8\ \ 6c)\ \ 19c\ \ 43$

$96\ \ 72c\ \ 19c\ \ 43$

$53\ \ 53c$

$c\ \ 1,\ g\ \ 2$

x-intercept $2\sqrt{g^2\ \ c}$ $2\sqrt{4\ \ 19}$ $2\sqrt{23}$

15.(B) Here we need to find out the locus of centroid first.

Let h,k be the centroid.

Hence, $3h\ \ a\ \sin t\ \cos t$

& $3k\ \ b\ \sin t\ \cos t$

$3h\ \ a^{\,2}\ \ \sin t\ \ \cos t^{\,2}$ ….(i)

$3k\ \ b^{\,2}\ \ \sin t\ \ \cos t^{\,2}$ …(ii)

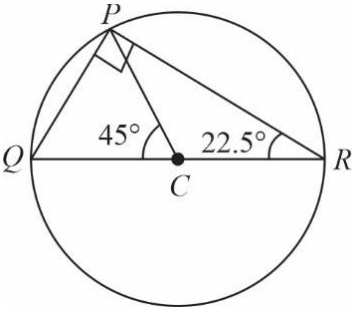
(i) + (ii) Ÿ $3h\ \ a^{\,2}\ \ 3k\ \ b^{\,2}\ \ 2$

Replacing h by x & k by y;

$\left(x\ \ \frac{a}{3}\right)^2 + \left(y\ \ \frac{b}{3}\right)^2 = \frac{2}{9}$ Ÿ centre $\left(\frac{a}{3}, \frac{b}{3}\right)$ $\left(1, \frac{1}{2}\right)$

Ÿ $a\ \ 3$ & $b\ \ 1$ Ÿ $a^2\ \ b^2\ \ 8$

16.(B) $x^2 + y^2 - x + 2y - \frac{11}{4} = 0$



$\therefore$ Radius $= \frac{1}{2}(PC + CQ + CR)$

$PQ = QR\sin 22.5 = 4\sin\left(\frac{\pi}{8}\right)$

$PR = QR\cos 22.5 = 4\cos\frac{\pi}{8}$

Area of $\Delta PQR = \frac{1}{2} PQ \cdot PR$

$\frac{1}{2}\left(16\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right)$

$4\sin\left(\frac{\pi}{4}\right)\cdot\frac{4}{\sqrt{2}} = 2\sqrt{2}$

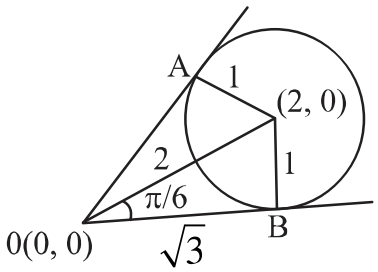
17.(C) $(D-0)^2 + (E-1)^2 = (E+1)^2$

$D^2 = 4E$

$x^2 = 4y$

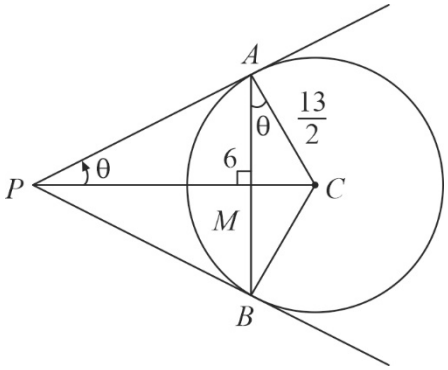
$A = 2\int_0^4 \left(4 - \frac{x^2}{4}\right)dx = \frac{64}{3}$

18.(B)



ar $\Delta OAB = \frac{1}{2}\cdot\sqrt{3}\cdot\sqrt{3}\sin\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{4}$

19 .(72)



AM MB 6

AC BC $\frac{13}{2}$

cos T $\frac{6}{\frac{13}{2}}$ $\frac{12}{13}$

sin T $\frac{5}{13}$

PM AM cos T

6 u$\frac{12}{5}$ Ÿ 5 PM 72

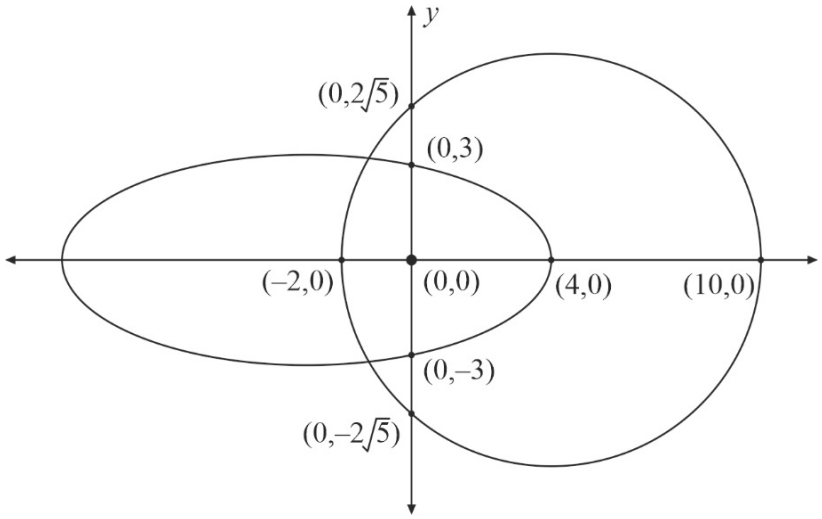
20.(27) S : $\frac{x\ 3^{2}}{16}$ $\frac{y\ 4^{2}}{9}$ d1; x, y •^1,2,3... `

T : x 7 ^{2} y 4 ^{2} d36; x, y •R

Let x 3 X & y 4 Y

S : $\frac{X^{2}}{16}$ $\frac{Y^{2}}{9}$ d1; X •^2, 1,0,1,..... `

T : X 4 ^{2} Y ^{2} d36; Y •^3, 2, 1,0,..... `



S ˆ T 2,0 , 1,0 ,....., 4,0 o7

1,1 , 0,1 ,..... 3,1 o5

1, 1 , 0, 1 ,..... 3, 1 o5

1,2 , 0,2 , 1,2 , 2,2 o4

1, 2 , 0, 2 , 1, 2 , 2, 2 o4

0,3 , 0, 3 o 2

Hence total points are 7 5 5 4 4 2

$\boxed{27}$

21 .(A) Equation of Circle

$(x \quad x_1)(x \quad x_2) \quad (y \quad y_1)y \quad y_2 \quad 0$

$x^2 \quad (x_1 \quad x_2)x \quad x_1x_2 \quad y^2 \quad (y_1 \quad y_2y) \quad y_1y_2 \quad 0$

$x^2 \quad 4x \quad 6 \quad y^2 \quad 2y \quad 7 \quad 0$

$x^2 \quad y^2 \quad 4x \quad 2y \quad 13 \quad 0$

a 2, b 1, c 13

a b c 12

22 .(A) Circle C_1 is $x \quad 2^2 \quad y^2 \quad 4$ Ÿ $x^2 \quad y^2 \quad 4x \quad 0$

$y \quad 2x$ meets the circle C_1 at 0,0 and $\left(\frac{4}{5}, \frac{8}{5}\right)$

Equation of tangent at C_2 at point A $\left(\frac{4}{5}, \frac{8}{5}\right)$ is $x \quad 2y \quad 4$

Which meets x-axis at P 4,0 and Q 0,2

Ÿ QA : AP 1 : 4

23 .(12) Image of centre c_1 1,3 in $x \quad y \quad 1 \quad 0$ is given by

$\frac{x_1 \quad 1}{1} \quad \frac{y_1 \quad 3}{1} \quad \frac{2 \quad 1 \quad 3 \quad 1}{1^2 \quad 1^2}$

Ÿ $x_1 \quad 2, y_1 \quad 2$

? Centre of circle c_2 { 2,2

? Equation of c_2 be $x^2 \quad y^2 \quad 4x \quad 4y \quad \frac{38}{5} \quad 0$

Now radius of c_2 is $\sqrt{4 \quad 4 \quad \frac{38}{5}} \quad \sqrt{\frac{2}{5}} \quad r$

radius of c_1^2 radius of c_2^2

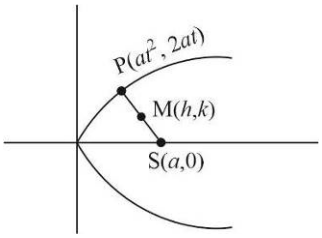
Ÿ $10 \quad D \quad \frac{2}{5}$ Ÿ $D \quad \frac{48}{5}$

? $D \quad 6r^2 \quad \frac{48}{5} \quad \frac{12}{5} \quad 12$

Solutions ofArchive - JEE Main & Advanced

Conic Sections	Class- XI | Mathematics

JEE Main 2021



1.(B) Let the midpoint be (h, k)

$? h = \frac{at^2 + a}{2}, k = \frac{2at + 0}{2} \quad ? t^2 = \frac{2h - a}{a} \& t = \frac{k}{a}$

$? \frac{k^2}{a^2} = \frac{2h - a}{a}$

? Locus of (h, k) is $y^2 = a(2x - a)$? Its directrix is $x = \frac{a}{2} - \frac{a}{2}$? $x = 0$

2.(B) Checking by options $\left(m = \frac{1}{\sqrt{3}}\right)$

$x^2 + 9y^2 = 9$

$2x + 18y\left(\frac{dy}{dx}\right) = 0 \quad Ÿ \quad \frac{dy}{dx} = -\frac{x}{9y} = -\frac{3\sqrt{3}}{2\left(\frac{1}{2}\right)} = -\frac{1}{\sqrt{3}}$

3.(D) $(t, t^2 - 4)$; Dist. From $y - 4x + 1 = 0$ is $\frac{t^2 - 4 - 4t + 1}{\sqrt{17}}$

For min. $\frac{d}{dt}(t^2 - 4t - 5) = 0$? point is (2, 8)

4.(B) $\frac{x^2}{a} - \frac{y^2}{b} = 1$ $\frac{x^2}{c} - \frac{y^2}{d} = 1$

$\left(\frac{dy}{dx}\right)_1 = \frac{x}{a}\cdot\frac{b}{y}$ $\left(\frac{dy}{dx}\right)_2 = \frac{x}{y}\cdot\frac{d}{c}$

Clearly $\left(\frac{dy}{dx}\right)_1 \cdot \left(\frac{dy}{dx}\right)_2 = 1$; $\frac{x^2}{y^2}\cdot\frac{bd}{ac} = 1$ ..(1)

Also $x^2\left(\frac{1}{a} - \frac{1}{b}\right) + y^2\left(\frac{1}{b} - \frac{1}{d}\right) = 0$

$\frac{x^2}{y^2} = \frac{\frac{1}{b} - \frac{1}{d}}{\frac{1}{a} - \frac{1}{c}}$ Ÿ $\frac{x^2}{y^2} = \frac{d - b}{c - a}\cdot\frac{ac}{bd}$ Ÿ $\frac{d - b}{c - a} = 1$; d b c a ; a b c d

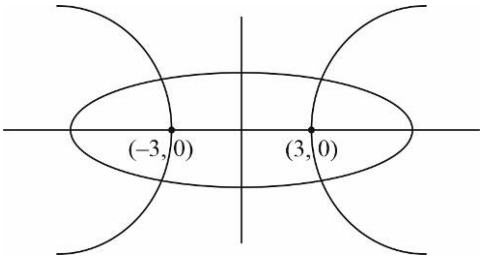
5.(A) $y = mx - \frac{a}{m}$ is tangent to $y^2 = 4ax$

i.e, $y = \frac{1}{2}x + 3$ is tangent to $y^2 = 6x$ So (5, 4) doesn't lie on it

6.(2) $\sqrt{3}kx - ky + 4\sqrt{3} = 0 \Rightarrow \frac{\sqrt{3}x - y}{k} = -4\sqrt{3} \Rightarrow 3x^2 + y^2 = 48 \Rightarrow \frac{x^2}{16} + \frac{y^2}{48} = 1$

$e = \sqrt{1 - \frac{48}{16}} = 2$

7.(A)



Equation of ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$

$a^2 = 25, b^2 = 16$

$\therefore b^2 = a^2(1 - e^2) \Rightarrow 16 = 25(1 - e^2)$

$e^2 = \frac{9}{25} \Rightarrow e = \frac{3}{5}$

? Eccentricity of Hyperbola $\frac{5}{3}$.

Let Eq^n of Hyperbola : $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1 \Rightarrow \frac{9}{A^2} = 1 \Rightarrow A^2 = 9.$

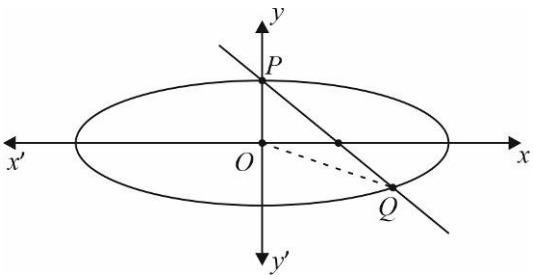
Also $B^2 = A^2(e^2 - 1)$

$B^2 = 9\left(\frac{25}{9} - 1\right)$

$B^2 = 16$

? Eq^n of Hyperbola : $\frac{x^2}{9} - \frac{y^2}{16} = 1$

8.(B)



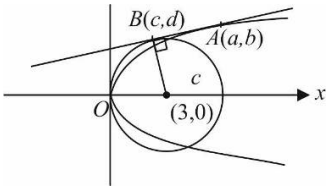
Making homogeneous the equation of ellipse $x^2 + 2y^2 = 2$ with the help of eq^n of line $x + y = 1$.

Ÿ x^2 $2y^2$ $2x$ y^2 0 Ÿ x^2 $4xy$ 0 Ÿ x^2 $4xy$ 0

Ÿ x x $4y$ 0 ? Lines are $x = 0$ and y $\frac{1}{4}x$.

? Angles between lines $\frac{S}{2}$ $\tan^{1}\frac{1}{4}$.

9.(9)



Let A $t^2, 2t$

? Equation of tangent at A

ty x t^2 Ÿ x ty t^2 0

' This is also the tangent to the circle. ? $\left|\frac{3\ 0\ t^2}{\sqrt{1\ t^2}}\right|$ 3 Ÿ t 0, $\sqrt{3}$, $\sqrt{3}$

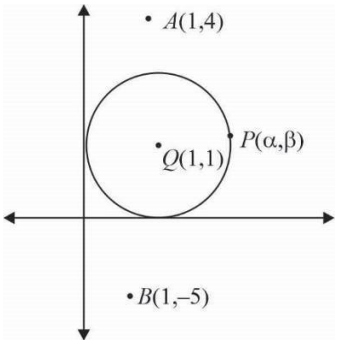
' Point lies on 1 st quadrant ? A $3, 2\sqrt{3}$

And Equation of tangent x $\sqrt{3}y$ 3 0

For point B, which is foot of perpendicular from (3,0) to the tangent x $\sqrt{3}y$ 3 0

$\frac{c\ 3}{1}$ $\frac{d\ 0}{\sqrt{3}}$ $\frac{3\ 0\ 3}{1\ 3}$? c $\frac{3}{2}, d$ $\frac{3\sqrt{3}}{2}$? 2 a c 2 § 3 © $\frac{3}{2}$ ' 9 ' 1

10.(A)



P D, E lies on x 1^2 y 1^2 1

PA^2 PB^2 D 1^2 E 4^2 D 1^2 E 5^2 2 D^2 E^2 2 D 2 E 43

2 2 E 1 2 E 43 { ' Point P D, E satisfies x 1^2 y 1^2 1 i.e.

D^2 E^2 2 D 2 E }

PA^2 PB^2 6 E 41

To get maximum value of PA^2 PB^2; E must be 2 & hence

P will be D, 2 Ÿ D 1. Hence, P will be 1, 2 . Hence A, P, B lie on a straight line.

11 .(3) $4x^2 \quad 9y^2 \quad 36 \;\ddot{Y}\; \frac{x^2}{9} \quad \frac{y^2}{4} \quad 1$

$x^2 \quad y^2 \quad \frac{31}{4}$

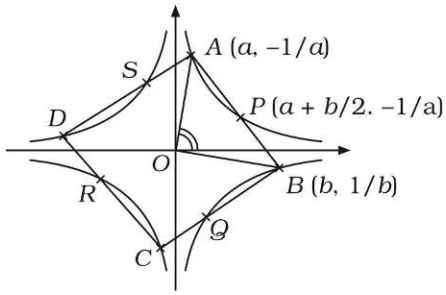
$9m^2 \quad 4 \quad \frac{31}{4} \; m^2 \quad 1$

$9m^2 \quad \frac{31}{4} m^2 \quad \frac{31}{4} \quad 4$

$\frac{5m^2}{4} \quad \frac{15}{4} \;\ddot{Y}\; m^2 \quad 3$

12 .(4) OA AOB

$\frac{1}{a^2} \; \frac{1}{b^2} \quad 1 \quad \ddot{Y} \quad a^2 b^2 \quad 1 \quad \ldots\ldots(i)$



$\frac{(a \quad b)^2}{4} \; \frac{\left(\frac{1}{b} \quad \frac{1}{a}\right)^2}{4} \quad 1$

$(a \quad b)^2 (a \quad b)^2 \quad 16a^2 b^2$

$16 \quad \ddot{Y} \quad (a^2 \quad b^2)^2 \quad 16$

If $a^2 \quad b^2 \quad 4$

$\ddot{Y} \quad a^2 \quad \frac{}{a^2} \quad 4 \quad \ddot{Y} \quad a^4 \quad 4a^2 \quad 1 \quad 0$

$\ddot{Y} \quad a^2 \quad \frac{4 \; r\sqrt{20}}{2} \;\ddot{Y}\; a^2 \quad 2 \quad \sqrt{5} \; ; \; b^2 \quad \frac{1}{2 \quad \sqrt{5}} \quad \sqrt{5} \quad 2$

$AB^2 \quad (a \quad b)^2 \quad \left(\frac{1}{b} \quad \frac{1}{a}\right)^2 \quad a^2 \quad b^2 \quad 2ab \quad \frac{1}{a^2} \quad \frac{1}{b^2} \quad \frac{2}{ab} \quad 2 \quad \sqrt{5} \quad \sqrt{5} \quad 2 \quad \sqrt{5} \quad 2 \quad \sqrt{5} \quad 2 \quad 4\sqrt{5}$

If $a^2 \quad b^2 \quad 4 \quad \ddot{Y} \quad a^2 \quad \frac{1}{a^2} \quad 4 \quad 0$

$a^4 \quad 4a^2 \quad 1 \quad 0$

$a^2 \quad \frac{4 \; r\sqrt{16 \quad 4}}{2}$

$a^2 \quad 2 \; r\sqrt{5}$

$a^2 \quad \sqrt{5} \quad 2$

$b^2 \quad \sqrt{5} \quad 2$

$AB^2 \quad (a \quad b)^2 \quad \left(\frac{1}{b} \quad \frac{1}{a}\right)^2 \quad a^2 \quad b^2 \quad 2ab \quad \frac{1}{a^2} \quad \frac{1}{b^2} \quad \frac{2}{ab} \quad \sqrt{5} \quad 2 \quad \sqrt{5} \quad 2 \quad \sqrt{5} \quad 2 \quad \sqrt{5} \quad 2 \quad 4\sqrt{5}$

Area of square $4\sqrt{5}$

Its square = 80

13.(C) Image of $y^2 \quad 4x$ about $y \quad x$ is $x^2 \quad 4y$

Tangent at $2,1$ is $x.x_1 \quad 2\,y \quad y_1$

$x.2 \quad 2\,y \quad 1$ is $x \quad y \quad 1$

14.(D) $\frac{x^2}{16} \quad \frac{y^2}{b^2} \quad 1$ and circle $x^2 \quad y^2 \quad 4b,\ b \quad !c_1$

Centre $0,0$ radius $2\sqrt{b}$

As $y^2 \quad 3x^2$ has intersection with both curves.

$\frac{x^2}{16} \quad \frac{3x^2}{b^2} \quad 1;\quad x^2\ b^2 \quad 48 \quad 16b^2$

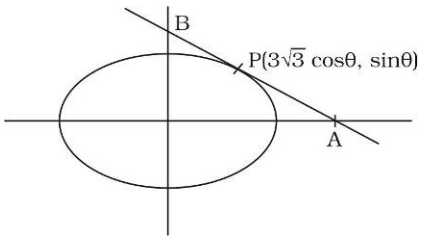
$x^2 \quad \frac{16b^2}{b^2 \quad 48};\ x^2 \quad 3x^2 \quad 4b;\ x^2 \quad b \quad \ddot{Y}\ \frac{16b^2}{b^2 \quad 48} \quad b \quad ;\qquad b \quad 12$

15.(D) Equation of tangent

$\frac{x}{3\sqrt{3}}\cos T \quad y\sin \quad T1$

A : $\left(\frac{3\sqrt{3}}{\cos T},0\right)$;

B : $\left(0,\frac{1}{\sin T}\right)$



Sum of intercept $\frac{3\sqrt{3}}{\cos T} \quad \frac{1}{\sin}$

$y \quad 3\sqrt{3}\sec T \quad \cos ec$

$y' \quad 3\sqrt{3}\sec T\tan \quad T\cos ec\cot T$

$y' \quad 0$

$3\sqrt{3} \quad u\frac{\sin T}{\cos T}\tan \quad T \cot$

$\tan T \quad \frac{1}{\sqrt{3}}$

$T \quad \frac{S}{6}$

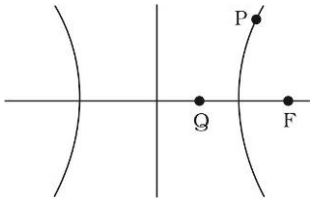
16.(D) $\frac{x^2}{4} \quad \frac{y^2}{2} \quad 1$

$a \quad 2 \qquad e^2 \quad 1 \quad \frac{1}{2} \quad \frac{3}{2} \ \ddot{Y}\ e \quad \sqrt{\frac{3}{2}}$

$b \quad \sqrt{2}$

Tangent equation at P : $4x \quad 2\,uy \quad u\sqrt{6} \quad 4$

$4x \quad 2\sqrt{6}y \quad 4$



Q : (1,0)

F : (ae,0) $\ddot{Y}$ $\sqrt{6},0$

LR at R

$x = ae = \sqrt{6}$

$4\sqrt{6} + 2\sqrt{6}\,y = 4$

$y = \dfrac{4 - \sqrt{6}}{2\sqrt{6}}$ $\quad R : \left(\sqrt{6}, \dfrac{2\sqrt{6} - 1}{\sqrt{6}}\right)$

Area of $\Delta = \dfrac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ \sqrt{6} & 0 & 1 \\ \sqrt{6} & \dfrac{2\sqrt{6}-1}{\sqrt{6}} & 1 \end{vmatrix}$

Area $= \dfrac{1}{2}\left|0 + \sqrt{6}\left(0 - \dfrac{2\sqrt{6}-1}{\sqrt{6}}\right) + 1\left(\dfrac{2\sqrt{6}-1}{\sqrt{6}} - 0\right)\right| = \left|\dfrac{\sqrt{6}-1}{\sqrt{6}} - \dfrac{\sqrt{6}-1}{1} - \dfrac{\sqrt{6}-1}{\sqrt{6}}\right| = \left|\dfrac{7-2\sqrt{6}}{\sqrt{6}}\right|$

17 .(C) For parabola $y^2 = 4ax$, for more than 3 normals from the point $(h, 0)$, we have

$h > 2a \Rightarrow a > 2 \cdot \dfrac{1}{2} \Rightarrow a > 1$

18 .(C) Tangent line to the parabola $y^2 = 4x + 20$ at (6,2) is $yy_1 = 2(x + x_1) + 20$ i.e. $2y = 2(x + 6) + 20$

i.e. $y = x + 4$ …(i)

This is also a tangent to the ellipse $\dfrac{x^2}{2} + \dfrac{y^2}{b} = 1$

$\Rightarrow y = x \pm \sqrt{2 + b}$ …(ii) {$m = 1$ using (i)}

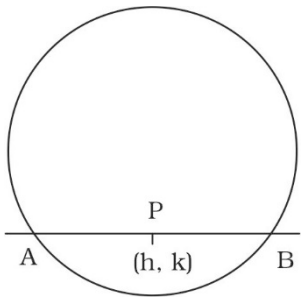
Comparing (i) & (ii) $\Rightarrow 4 = \pm\sqrt{2 + b} \Rightarrow 16 = 2 + b \Rightarrow b = 14$

19 .(A) Let mid-point of chord AB of the circle is $P(h, k)$

$\therefore$ Equation of AB is $T = S_1$

$\Rightarrow hx + ky = h^2 + k^2$

$\Rightarrow y = -\dfrac{h}{k}x + \dfrac{h^2 + k^2}{k}$ …(i)



This is tangent to the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

$\Rightarrow c^2 = a^2m^2 - b^2 \Rightarrow \left(\dfrac{h^2 + k^2}{k}\right)^2 = 9\left(-\dfrac{h}{k}\right)^2 - 16$

$\therefore$ Locus is $(x^2 + y^2)^2 = 9x^2 - 16y^2 = 0$

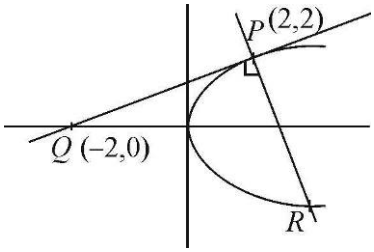
20 .(D) Equation of tangent at P is

x 2y 2 0

? Q 2,0

Now, 2at$_1$ 2 Ÿ t$_1$ 2

Again t$_2$ t$_1$ $\frac{2}{t_1}$ 3



? R { $\frac{1}{2}$ 3 ^{2} , 2 $\frac{1}{2}$, 3 , { $\frac{9}{2}$, 3

? ar PQR $\frac{1}{2}\left\|\begin{matrix} 2 & 2 & 1 \\ 2 & 0 & 1 \\ \frac{9}{2} & 3 & 1 \end{matrix}\right\|$ $\frac{25}{2}$ sq.units

21 .(1.25) $|D \quad E|$ J1.25

22 .(34) ' y^2 64x Ÿ focus{ 16,0

y mx c is a focal chord.

Y c 16m …(i)

' y mx c is tangent to x 10 ^{2} y^2 4 Ÿ y m x 10 r 2$\sqrt{1 \quad m^2}$

Ÿ c 10m r 2$\sqrt{1 \quad m^2}$ Ÿ 16m 10m r 2$\sqrt{1 \quad m^2}$ Ÿ m $\frac{1}{2\sqrt{2}}$ m! 0

Y c $\frac{8}{\sqrt{2}}$? 4$\sqrt{2}$(m c) 4$\sqrt{2}$ u$\frac{17}{2\sqrt{2}}$ 34

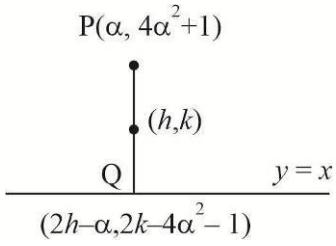
23 .(D) 2h D 2k 4 D^2 1

h k 4 D^2 D1 … (1)

2 k h 4 D^2 D1 … (2)

Slope of PQ is – 1

$\frac{k \quad 4 D^2 \quad 1}{h \quad D}$ 1



k 4 D^2 1 h k ; h k 4 D^2 D1

(1) & (2) 3h k 2 D

D $\frac{3h \quad k}{2}$

Putting in equation (1)

h k 4 $\frac{3h \quad k}{2}$ ^{2} , $\frac{3h \quad k}{2}$ 1 ; 3h k ^{2} $\frac{3k}{2}$ $\frac{h}{2}$ 1 Ÿ 2 3x y ^{2} 3y x 2

24 .(9) $D, \sqrt{6}\ D$ is a point on parabola

$D^2 \quad 3 \quad D^2 \quad \frac{3}{2} \quad \sqrt{6}\ D$

$D^2 \quad 9 \quad D^2 \quad 6 \quad D \frac{9}{4} \quad 6 \quad D 3\sqrt{6}$

$\frac{d}{d\ D} D^2 \quad 2\ D \quad 6 \quad 6 \quad 3\sqrt{6} \quad \frac{1}{2\sqrt{D}} \quad 0$

$2\ D \quad \frac{3\sqrt{6}}{2\sqrt{D}}; \qquad D\sqrt{D} \quad \frac{3\sqrt{6}}{4}$

$D^3 \quad \frac{9}{16} \quad 6 \quad \frac{27}{8} \quad \ddot{Y} \quad D \frac{3}{2}$

$E \quad \sqrt{6}\ D \quad \sqrt{\frac{6\ 3}{2}} \quad 3; \quad 2\ D \quad E \quad 2 \quad \frac{3}{2} \quad 3 \quad 9$

25 .(B) Since, tangent to the hyperbola $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$ with slope 'm' is

$y \quad mx \quad r\sqrt{a^2 m^2 \quad b^2} \quad \ddot{Y} \qquad y \quad mx \quad r\sqrt{3m^2 \quad 3}$ is the tangent for $\frac{x^2}{3} \quad \frac{y^2}{3} \quad 1$

Comparing it with $2x \quad y \quad k$ i.e. $y \quad 2\ x \quad k$

$\ddot{Y} \qquad m \quad 2\ \&\ k \quad r\sqrt{3 \quad 2^2 \quad 3} \quad 3 \quad 3 \quad 'k \quad 0$

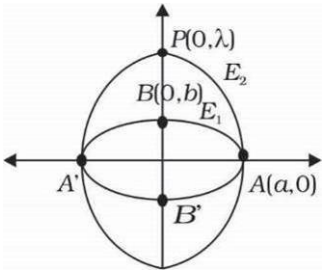
So, equation of tangent is $2x \quad y \quad 3$ i.e. $y \quad 2x \quad 3$

This is also tangent at $y^2 \quad Dx \quad 4 \quad \frac{D}{4} \quad x; \quad a \quad \frac{D}{4} \qquad \ddot{Y} \qquad y \quad mx \quad \frac{a}{m} \quad mx \quad \frac{D}{4m}$

Compare it with $y \quad 2x \quad 3$; we get $m \quad 2\ \&\ \frac{D}{4 \quad 2} \quad 3$

i.e. $D \quad 24$

26 .(B)



$E_1 : \frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1; \quad a \quad !b \quad !0, \quad E_2 : \frac{y^2}{O^2} \quad \frac{x^2}{a^2} \quad 1; \quad O \quad a \quad 0$

Foci of E_2 are $0, rb \quad \{ 0, r\ e \quad \ddot{Y} b \quad eC$ …(i)

Let 'e' be the eccentricity of E_1 & E_2.

Ÿ $e^2 \quad 1 \quad \frac{b^2}{a^2}$ & $e^2 \quad 1 \quad \frac{a^2}{2}$ …(ii)

Ÿ $1 \quad \frac{b^2}{a^2} \quad 1 \quad \frac{a^2e^2}{b^2}$ {using (i)}

Ÿ $\frac{b^4}{a^4} \quad e^2$ Ÿ $e \quad \frac{b^2}{a^2}$ …(iii)

(iii) in (ii) Ÿ $e^2 \quad 1 \quad e$ Ÿ $e^2 \quad e \quad 1 \quad 0$ Ÿ $e \quad \frac{1 \quad r\sqrt{5}}{2}$ Ÿ $e \quad \frac{\sqrt{5} \quad 1}{2}$

27 .(D) For ellipse

$b^2 \quad a^2 \quad 1 \quad e^2 \quad a^2 \quad \frac{1}{3} \quad \frac{2a^2}{3}$

$2a^2 \quad 3b^2$ …….(i)

And it pass through $\frac{\sqrt{3}}{2}, 1$

$\frac{3/2}{a^2} \quad \frac{1}{b^2} \quad 1$

$3b^2 \quad 2a^2 \quad 2a^2b^2$ ……(ii)

From (i) and (ii) $a^2 \quad 3, b^2 \quad 2$

Focus $F \quad ae, 0 \quad 1, 0$

Let the point $a\cos T, b\sin T$ lie at the circle and ellipse both

$a\cos T \quad 1^2 \quad b^2\sin^2 T \quad r^2$

$\sqrt{3}\cos T \quad 1^2 \quad 2\sin^2 T \quad 4/3$

$3\cos^2 T \quad 2\sqrt{3}\cos T \quad 1 \quad 2\sin^2 T \quad \frac{4}{3}$

$\cos^2 T \quad 2\sqrt{3}\cos T \quad \frac{5}{3} \quad 0$

$3\cos^2 T \quad 6\sqrt{3}\cos T \quad 5 \quad 0$

$\cos T \quad \frac{1}{\sqrt{3}}$ or $\frac{5}{\sqrt{3}}$

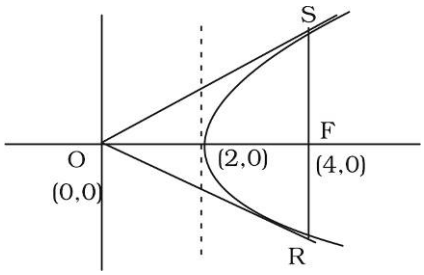
$\cos T \quad \frac{1}{\sqrt{3}}$ Ÿ $\sin T \quad \sqrt{\frac{2}{3}}$

$P \quad a\cos T, b\sin T, Q \quad a\cos, T \quad b\sin$

$PQ \quad 2b|\sin T| \quad 2.\sqrt{2} \quad \frac{\sqrt{2}}{\sqrt{3}} \quad \frac{4}{\sqrt{3}}$

$PQ^2 \quad \frac{16}{3}$

28 .(C) vertex 2,0



Focus = (4,0)

Equation of parabola $y^2 = 8x$

$a = 2$

'SOR = 2 ' ORS = 2 $\frac{1}{2}$ OF. FS = 4 u4 = 16 units

29 .(B) Given equation can be written as $\frac{(x-1)^2}{9} - \frac{(y-2)^2}{16} = 1$ or $\frac{X^2}{9} - \frac{Y^2}{16} = 1$ where

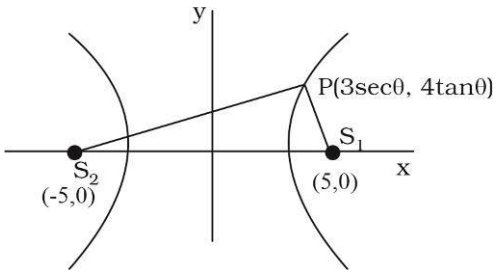
$x - 1 = X$ and $y - 1 = Y$

Eccentricity of the hyperbola is

$e = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

X coordinate of foci are r5

Y coordinate of foci is 0



Let (h, k) be the coordinate of the centroid of the triangle.

$h = \frac{3\sec\theta + 5 - 5}{3} \Rightarrow \sec\theta = h$

$k = \frac{4\tan\theta + 0 - 0}{3} \Rightarrow \tan\theta = \frac{3k}{4}$

Now $\sec^2\theta - \tan^2\theta = 1$

$h^2 - \frac{9}{16}k^2 = 1$

$X^2 - \frac{9}{16}Y^2 = 1$

$16(x-1)^2 - 9(y-2)^2 = 144$

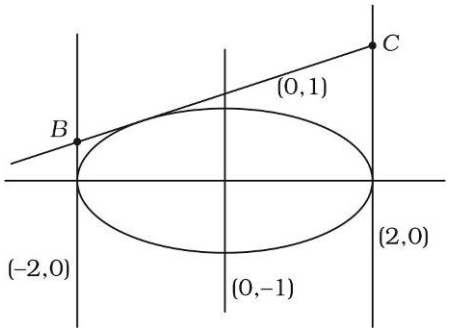
$16x^2 - 9y^2 - 32x + 36y - 36 = 0$

30 .(D) $\frac{x^2}{4} \quad \frac{y^2}{1} \quad 1$

Let the tangent be

$\frac{x}{2}\cos T \; y \sin \quad T1$

B $\{(\ 2, y_1)$ and C $\{(2, y_2)$



Equation of the circle with BC as diameter is

$(x \quad 2)(x \quad 2) \quad (y \quad y_1)(y \quad y_2) \quad 0$

$x^2 \quad y^2 \quad (y_1 \quad y_2)y \quad y_1 y_2 \quad 4 \quad 0$

$\cos T \; y_1 \sin \quad T1 \qquad \ddot{Y} \qquad y_1 \quad \frac{1 \quad \cos T}{\sin T} \quad \frac{2\cos^2 /2T}{2\sin /2T\cos /2 \; T} \quad \cot T/2$

$\cos T \; y_2 \sin \quad T1 \qquad \ddot{Y} \qquad y_2 \quad \tan T/2$

Equation of the circle

$(x^2 \quad y^2 \quad 3) \quad \left(\cot \frac{T}{2} \quad \tan \frac{T}{2}\right) y \quad 0$

Which always passes through $(\sqrt{3},0)$ and $(\ \sqrt{3},0)$

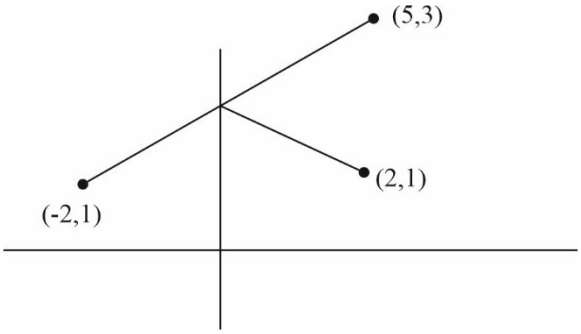
31 .(C) Equation of reflected ray

$y \quad 1 \quad \frac{3 \quad 1}{5 \quad 2}(x \quad 2)$

$\ddot{Y} \quad 7(y \quad 1) \quad 2(x \quad 2)$

$\ddot{Y} \quad 7y \quad 7 \quad 2x \quad 4$

$\ddot{Y} \quad 2x \quad 7y \quad 11 \quad 0$



Given, $\frac{a}{e} \quad ae \quad \frac{8}{\sqrt{53}}$

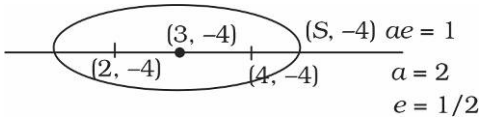
$\ddot{Y} \quad 3a \quad \frac{a}{3} \quad \frac{8}{\sqrt{53}} \quad \ddot{Y} \quad a \quad \frac{3}{\sqrt{53}}$

Other directrix $2x \quad 7y \quad c \quad 0$

Such that $\left|\frac{c \quad 11}{\sqrt{53}}\right| \quad \frac{2 \; u9}{\sqrt{53}} \quad \ddot{Y} \quad c \quad 11 \; r \; 18 \quad 29, \quad 7$

$2x \quad 7y \quad 29 \quad 0 \; or \; 2x \quad 7y \quad 7 \quad ($

32 .(3) $\frac{(x \quad 3)^2}{4} \quad \frac{(y \quad 4)^2}{3} \quad 1 \quad \ddot{Y} \quad \frac{(x \quad 3)^2}{4} \quad \frac{m^2x^2}{3} \quad 1$



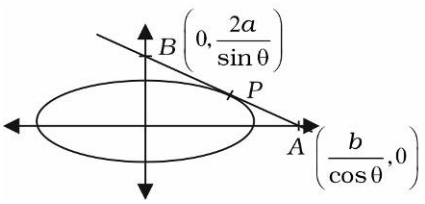
$y \quad mx \quad 4$

$x^2(3 \quad 4m^2) \quad 18x \quad 27 \quad 12 \quad 0$

$x^2(3 \quad 4m^2) \quad 18x \quad 15 \quad 0$

D 0 Ÿ $m^2 \frac{3}{5}$ Ÿ $5m^2$ 3

33 .(2) Equation of tangent at P is



$\frac{x\cos T}{b} \frac{y\sin T}{2a}$ 1

? A $\{ \frac{b}{\cos T}, 0$, B $\{ 0, \frac{2a}{\sin T}$, Ÿ area of ' OAB $\frac{1}{2} u \frac{b}{\cos T} u \frac{2a}{\sin T} \frac{2ab}{\sin}$

? Minimum are of 'OAB 2ab Ÿ K 2

34 .(D) Equation of tangent at (2, 4) is x y 2 0 ...(i)

And equation of normal at (2, 4) is x y 6 0 …(ii)

And Equation of directrix is x 2 …(iii)

? A { 2,0 , B { 2, 8

And Q { a,b and P 2, 4 and AQBP is a square

Ÿ Mid -point of AB = Mid -point of PQ

Ÿ AB 2, 4 $\frac{a\ 2}{2}, \frac{b\ 4}{2}$, Ÿ a 6, b 4 ? 2a b 16

35 .(A) ' x^2 $9y^2$ 4x 3 0 Ÿ x^2 4x 4 $3y^2$ 1

Ÿ $\frac{x\ 2^2}{1} \frac{y^2}{\frac{1}{3}}$ 1 (Equation of an Ellipse)

As it represent an equation of ellipse x and y can very on or inside the ellipse.

Ÿ x 2 • [-1,1] and y • $\left[-\frac{1}{3}, \frac{1}{3}\right]$ Ÿ x • [1,3] and y • $\left[-\frac{1}{3}, \frac{1}{3}\right]$

36 .(D) $\frac{24}{a^2} \frac{3}{b^2}$ 1 and $\frac{5}{4}$ 1 $\frac{b^2}{a^2}$ Ÿ $\frac{b^2}{a^2} \frac{1}{4}$ or 2b = a

Solve to get b $\sqrt{3}$, a $2\sqrt{3}$

So, the hyperbola is $\frac{x^2}{12} \frac{y^2}{3}$ 1

Ÿ Tangent at ($2\sqrt{6}$, $\sqrt{3}$) is - $\frac{\sqrt{6}x}{6} \frac{\sqrt{3}y}{3}$ 1 and normal is $\frac{12x}{2\sqrt{6}} \frac{3y}{\sqrt{3}}$ 15

Ÿ Q {(0, $\sqrt{3}$), R {(0,5 $\sqrt{3}$) Ÿ QR $6\sqrt{3}$

37.(C) Let the mid -point be (h, k) so the equation is:

xh yk h 2 k 2 or

$y \quad \frac{xh \quad (h^2 \quad k^2)}{k} \quad x \quad \frac{h}{k} \cdot \frac{(h^2 \quad k^2)}{k}$

This is tangent to $y^2 \quad 8x$ is: $y \quad mx \quad \frac{2}{m}$ Ÿ $m \quad \frac{h}{k}$ and $\frac{(h^2 \quad k^2)}{k} \quad \frac{2}{m}$

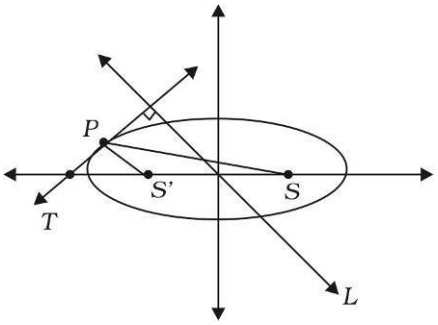
$2 \quad \frac{(h^2 \quad k^2)}{k^2}.h$

$2y^2 \quad x^3 \quad xy^2$ or $y^2(x \quad 2) \quad x^3$

38 .(B) $\frac{x^2}{8} \quad \frac{y^2}{4} \quad 1$ .....(i)

$a^2 \quad 8; b^2 \quad 4$

$b^2 \quad a^2 \quad 1 \quad e^2$



Ÿ $e^2 \quad 1/2$ Ÿ $e \quad \frac{1}{\sqrt{2}}$

S ae,0 { S 2,0 & S 2,0

Let T be the tangent at a point lies in the 2nd quadrant on ellipse and L be the line $x \quad 2y \quad 0$ such that T AL. Now; $T : y \quad mx \quad r\sqrt{a^2m^2 \quad b^2}$ 2x r6 ^T A `

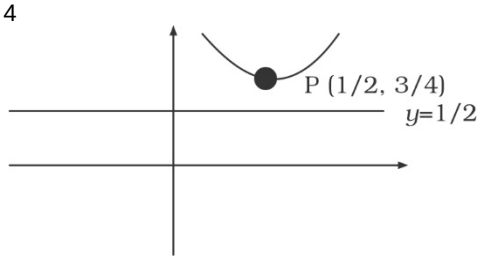
Since, P lies in 2nd quadrant Ÿ $T : y \quad 2x \quad 6$ .....(ii)

(i) & (ii) Ÿ P 8/3,2/3 . Now; Ar. 'PSS' A $\frac{1}{2}\begin{vmatrix} 2 & 0 & 1 \\ 2 & 0 & 1 \\ 8/3 & 2/3 & 1 \end{vmatrix}$ 4/3

Now; 5 e^2 A $\frac{1}{2} \quad \frac{4}{3}$ 6

39 .(C) Equation of parabola X^2 Y where X x $\frac{1}{2}$

Y y $\frac{3}{4}$



Line x $\frac{1}{2}$ meets the parabola

X^2 Y at P(1,1)

For x^2 y at P(1,1)

Equation of normal { X 2y 3 0

Solving with X^2 Y, we get Q $\left(-\frac{3}{2}, \frac{9}{4}\right)$? PQ^2 $\frac{125}{16}$

40 .(A) Point of intersection $\left(r\frac{3}{2}, r\frac{\sqrt{3}}{2}\right)$

Slope of tangent drawn to ellipse m_1

m_1 $\frac{dy}{dx}$ $\frac{x_1}{9y_1}$ $\frac{1}{3\sqrt{3}}$

Slope of tangent drawn to circle m_2

m_2 $\frac{dy}{dx}$ $\frac{x_1}{y_1}$ $\sqrt{3}$

Now tan T 2/ 3

41 .(C) $144x^2$ $25y^2$ 3600

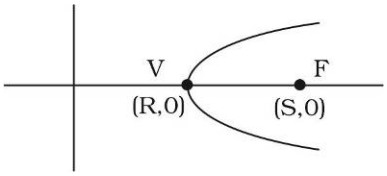
12 x cos T 5 y cos T 60

$\frac{x\cos T}{5}$ $\frac{y\sin T}{12}$ 1

Equation $\frac{x\cos T}{a}$ $\frac{y\sin T}{b}$ 1 is tangent to $\frac{x^2}{a^2}$ $\frac{y^2}{b^2}$ 1

$\frac{x^2}{25}$ $\frac{y^2}{144}$ 1 Ÿ $144x^2$ $25y^2$ 3600

42 .(A)



Latus rectum 4(S R)

43 .(36) Normals are to be drawn at

$A(\sec T, 2\tan)T$ and $B(\sec I, 2\tan)I$ on the curve $2x^2 \quad y^2 \quad 2$.

But the points do not lie at the given curve as per parametric point the co -ordinate must be in the form of $(a\sec D, b\tan)D$

i.e., $(\sec D, \sqrt{2}\tan)D$.

The data is the question is wrong so it cannot be solved further.

44 .(A) As equation of tangents are perpendicular

Ÿ Their point of intersection lies at directrix that is $x \quad 3 \quad 4$ Ÿ $x \quad 1 \quad 0$

JEE Advanced 2021

1 .(ABD) P lies on directrix $x \quad 2$

(A) Correct using the property that portion of the tangent cut off between directrix and the point of contact subtends right angle at the focus.

(B) Correct using the property that tangents to a parabola from any point lying on its directrix are perpendicular to each other

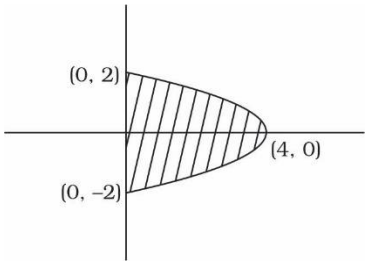
(C) $F : (2, 0)$ Ÿ $PF \quad 4\sqrt{2}$

(D) Correct using the property that points of contact of perpendicular tangents of a parabola are end -points of its focal chord.

2 .(1.50)

3 .(2)

Let the centre of the circle in family F with largest radius be $(h, 0)$ and let its radius C



Ÿ Equation of circle is $(x \quad h)^2 \quad y^2 \quad O^2$

This will touch the parabola $y^2 \quad 4 \quad x$

Y $(x \quad h)^2 \quad 4 \quad x \quad O$ has equal roots Y $O^2 \quad 15/4 \quad h$

Since the given circle lies inside the region R:

$0 \quad dh \quad O$ and $h \quad O4$

Ÿ $4O^2 \quad 4 \quad O15 \quad 0d$ and $\frac{15}{4} \quad {}^2 \quad O \quad 4C$ Ÿ $4O^2 \quad 4 \quad O15 \quad 0d$ and ${}^2 \quad O \quad \frac{1}{4}O \quad 0$

Y $O \bullet^a \neg 5/2, 3/2$ Y $O_{max} \quad 3/2$ and $h \quad 3/2$

So, the required circle is $\left(x \quad \frac{3}{2}\right)^2 \quad y^2 \quad \left(\frac{3}{2}\right)^2$ and it touches the given parabola $y^2 \quad 4 \quad x$ at $(2, r\sqrt{2})$ Ÿ $D \quad 2$

NOTE: One can verify that any general point on the circle $\left(\frac{3}{2}(1 \quad \cos T), \frac{3}{2}\sin T\right)$ will lie inside the region R.

4 .(4) In 'PQQ', using mid -point theorem, line segment joining M(P, Q) and M(P, Q') is parallel to QQ' and is equal to half its length

Ÿ Max length of the line segment $\frac{1}{2}$ u Max length of the line segment QQ' $\frac{1}{2}$ u length of longest Chord of E $\frac{1}{2}$ u length of major axis of E = 4

Solutions of Archive - JEE Main & Advanced

Conic Sections	Class - XI | Mathematics

JEE Main 2022

1.(A) Let point P(x,y)

$Y - y = y'(X - x)$

$Y - 0 = X - x - \frac{y}{y'}$

$Q \left(x - \frac{y}{y'}, 0\right)$

Mid-point of PQ lies on y axis

$x - \frac{y}{y'} + x = 0$

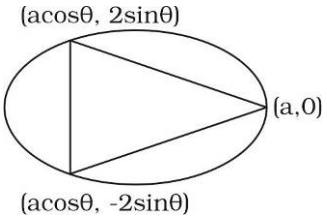
$y' - \frac{y}{2x} = 2\frac{dy}{y} - \frac{dx}{x} = 2\ln y - \ln x = \ln c$

$y^2 = cx$

' this curve passes through (3,3) ? c = 3

? required parabola $y^2 = 3x$ and L.R. = 3

2.(A) $\frac{x^2}{a^2} + \frac{y^2}{4} = 1,\ a > 2$



Area $= \frac{1}{2}a(1 + \cos\theta)(4\sin\theta)$

Area $= 2a(1 + \cos\theta)\sin\theta$

$\frac{dA}{d\theta} = 2a(\sin^2\theta + \cos\theta + \cos^2\theta) = 0$

? $1 + \cos\theta + 2\cos^2\theta = 0$

? $\cos\theta = 1$ (not possible in this case)

and $\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$

$\frac{d^2A}{d\theta^2} = 2a(2\sin 2\theta + \sin\theta)$

$\frac{d^2A}{d\theta^2} < 0$ for $\theta = \frac{2\pi}{3}$

$\Rightarrow A_{max} = \frac{3\sqrt{3}}{2}a = 6\sqrt{3} \Rightarrow a = 4$

$\Rightarrow$ eccentricity $e = \sqrt{\frac{a^2 - b^2}{a^2}} = \frac{\sqrt{3}}{2}$

3.(42) $\frac{x^2}{a^2} - \frac{y^2}{1} = 1$ $\frac{x^2}{4} - \frac{y^2}{3} = 1$

$e_H = \sqrt{1 + \frac{1}{a^2}}$ $e_E = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

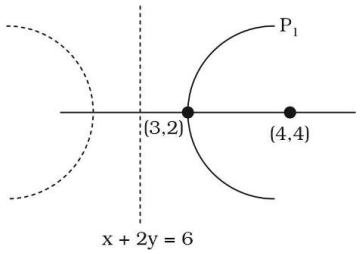
L.R. (Hyperbola) $= \frac{2}{a}$ L.R. $= 2 \times \frac{3}{2} = 3$

$\Rightarrow \frac{2}{a} = 3$ $\Rightarrow a = \frac{2}{3}$

$e_H = \sqrt{1 + \frac{1}{(2/3)^2}} = \frac{\sqrt{13}}{2}$

$\Rightarrow 12\, e_E^2 - e_H^2 = 12\left(1 - \frac{13}{4}\right) = -42$

4.(10) Focus $(4,4)$ and vertex $(3,2)$



$\Rightarrow$ Point of intersection of directrix with axis of parabola $= A\ (2,0)$

The directrix of P_1 is $x - 2y = 2$

Image of $x - 2y = 2$ w.r.t line $x - 2y = 6$ is $x - 2y = 10$

5.(A) $(x-2)^2 + (y-4)^2 = \lambda(4x + y - 4) = 0$

As circle passing through (0,6) therefore,

$4 + 4 = \lambda(6 - 4) = 0 \Rightarrow \lambda = \frac{8}{10} = \frac{4}{5}$

$\Rightarrow (x^2 - 4 - 4x) + (y^2 - 16 - 8y) = \frac{4}{5}(4x + y - 4) = 0$

$\Rightarrow x^2 + y^2 - \frac{4x}{5} - \frac{4y}{5} - \frac{84}{5} = 0$

$\Rightarrow A + C = \frac{80}{5} = 16$

6.(D) $a^2x^2 - \left(\frac{x}{2} - 1\right)^2 = b^2$

Discriminant = 0 $\Rightarrow (4a^2 - 1)x^2 - 2x + 1 - 4b^2 = 0$

$4 - 4(4a^2 - 1)(1 - 4b^2) = 0$

$4 - 4b^2 = 4a^2 - 16a^2b^2$

$\frac{1}{a^2} + \frac{4}{b^2} = 4$

7.(2929) Equation of tangent of parabola $y^2 = 4x$ is $y = mx + \frac{1}{m}$

$\left(-\frac{1}{5}\cos\theta, \frac{1}{2}\sin\theta\right) \equiv (\alpha, \beta)$

$\therefore$ equation of tangent is

$\frac{1}{2}\sin\theta = m\left(-\frac{1}{5}\cos\theta\right) + \frac{1}{m}$

$m^2\left(\frac{\cos\theta}{5}\right) - m\left(\frac{1}{2}\sin\theta\right) + 1 = 0$

$\therefore m_1 + m_2 = \frac{\frac{1}{2}\sin\theta}{\frac{\cos\theta}{5}}$ ...(i)

And $m_1m_2 = \frac{5}{\cos\theta}$ ...(ii)

From (i) and (ii)

$\cos\theta = \frac{5}{2}\sqrt{2}a \quad \Rightarrow \beta = \frac{5\sqrt{2}a}{10}$

$(10\beta - 5)^2 = 29$ ...(i)

$\alpha^2 = \frac{1}{4}\sin^2\theta = \frac{16\beta^2 - 50\beta + 29}{4}$

$\therefore (10\beta - 5)^2 = (16\beta^2 - 50\beta + 29)^2 = 2929$

8.(C) $4 - kx + x = x^2$

$\Rightarrow x^2 - (k - 1)x + 4 = 0$

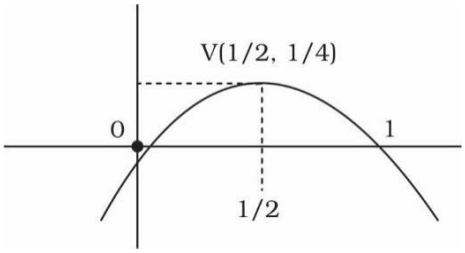
$\Rightarrow D = 0 \Rightarrow k = 5$

$(k - 1)^2 - 16 = 0$

$k = 5$

$\Rightarrow P \equiv (2, 6)$

$V \equiv \left(\frac{1}{2}, \frac{1}{4}\right)$



Slope of $PV = \frac{5}{2}$

9.(A) Let P (x_1, y_1); Q (x_2, y_2)

Ÿ $\frac{x^2}{4}$ $\frac{(x \quad 1)^2}{2}$ 1

Ÿ $3x^2$ $4x$ 2 0 $\left[\begin{array}{l} x_1 \\ x_2 \end{array}\right.$

r $\frac{1}{2}\sqrt{(x_1 \quad x_2)^2 \quad (y_1 \quad y_2)^2}$

$\frac{1}{2}$ u$\sqrt{2}\sqrt{(x_1 \quad x_2)^2}$ $\frac{1}{\sqrt{2}}\sqrt{(x_1 \quad x_2)^2 \quad 4x_1x_2}$

r $\frac{1}{\sqrt{2}}$ u$\sqrt{\frac{40}{9}}$

10.(85) $\frac{x^2}{a^2}$ $\frac{y^2}{b^2}$ 1, e $\frac{5}{4}$

$\frac{a^2x}{x_1}$ $\frac{b^2y}{y_1}$ a^2 b^2 Ÿ $\frac{\sqrt{4}a^2x}{8}$ $\frac{5b^2y}{12}$ a^2 u$\frac{25}{16}$ Ÿ $\frac{\sqrt{5}x}{8}$ $\frac{5}{12}$ $\frac{b^2}{a^2}$y $\frac{25}{16}$

e^2 1 $\frac{b^2}{a^2}$ $\frac{25}{16}$

$\frac{b^2}{a^2}$ $\frac{9}{16}$

Ÿ $\frac{2\sqrt{5}x}{2 \text{ u}8}$ $\frac{5}{12}$ u$\frac{9}{16}$y $\frac{25}{16}$

Ÿ $2\sqrt{5}x$ $\frac{16y}{4}$ 25 O 100, E15

Ÿ $8\sqrt{5}x$ 15y 100 O E85

11.(C) y m_1x c_1; y m_2x c_2.

Tangent to y^2 x Ÿ y mx $\frac{1}{4m}$ …. (i)

Tangent to x^2 y^2 2.

Then c^2 r^2 1 m^2

$\frac{1}{16m^2}$ 2 1 m^2

m^4 m^2 $\frac{1}{32}$ 0

§ :m^2 $\frac{1}{2}$ ^{2} ; $\frac{1}{32}$ $\frac{1}{4}$ $\frac{1 \quad 8}{32}$ $\frac{9}{32}$
©

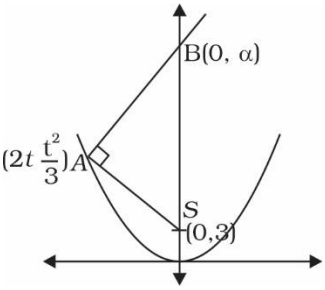
m^2 $\frac{1}{2}$ r$\frac{3}{4\sqrt{2}}$ Ÿ m^2 $\frac{3}{4\sqrt{2}}$ $\frac{1}{2}$

m^2 $\frac{3 \quad 2\sqrt{2}}{4\sqrt{2}}$

$8\left|m_1 m_2\right| \quad 8 \; u \; \frac{3 \quad 2\sqrt{2}}{4\sqrt{2}} \; , \; \frac{6 \quad 4\sqrt{2}}{\sqrt{2}} \; u \; \frac{\sqrt{2}}{\sqrt{2}}$

$3 \quad 2\sqrt{2} \; . \; \sqrt{2} \quad 3\sqrt{2} \quad 4$

12.(D)



Parabola $x^2 \quad 12y$

SA ASB

So, $m_{AS} . m_{AB} \quad 1$

$\frac{3 \quad \frac{t^2}{3}}{0 \quad 2t} . \frac{D \quad \frac{t^2}{3}}{0 \quad 2t} \; , \; 1$

By solving $3 \; D \; \frac{27t^2 \quad t^4}{t^2 \quad 9}$

Ordinate of centroid of 'SAB $K \; \frac{D \; \frac{t^2}{3} \quad 3}{3}$

$k \; \frac{9 \quad 3 \; D \; t^2}{9}$; $\lim_{t \; o 1} k \quad \lim_{t \; o 1} \frac{1}{9} \; t^2 \; \frac{27t^2 \quad t^4}{t^2 \quad 9} \; , \; \frac{13}{18}$

13.(B) $y \quad mx \quad \sqrt{16m^2 \quad 9}$ is a tangent to $x^2 \quad y^2 \quad 12$

Ÿ $\left|\frac{\sqrt{16m^2 \quad 9}}{\sqrt{m^2 \quad 1}}\right| \quad \left|\sqrt{12}\right|$

Ÿ $16m^2 \quad 9 \quad 12(m^2 \quad 1)$

Ÿ $4m^2 \quad 3$ Ÿ $12m^2 \quad 9$

14.(C) $h \; \frac{4 \quad 2\cos \; T}{2}$; $k \; \frac{3 \quad \sqrt{2}\sin}{2}$

Ÿ $\frac{2h \quad 4}{2}^2 \; , \; \frac{2x \quad 3}{\sqrt{2}}^2 \; , \; 1$ Ÿ $(x \quad 2)^2 \quad 2(y \quad 3/2)^2 \quad 1$

Ÿ $e^2 \quad 1 \quad \frac{1}{2} \quad \frac{1}{2}$ Ÿ $e \quad \frac{1}{\sqrt{2}}$

15.(C) $\left(\frac{8}{a}\right)^2 \cdot 3 \quad 1$ Ÿ $\left(\frac{8}{a}\right) \cdot 2$ Ÿ $a \quad 4$

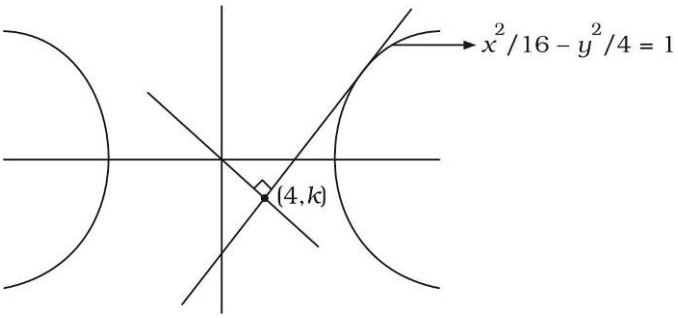
Now, $\frac{x^2}{16} \quad \frac{y^2}{9} \quad 1$

$\frac{x}{8} \quad \frac{2y}{9} y \; c \; 0$ Ÿ $y \; c \; \frac{9}{16}\frac{x}{y}$

$\frac{1}{y \; c} \quad \frac{16y}{9x}$

Normal $y \quad 3\sqrt{3} \quad \frac{2}{9}(3\sqrt{3})(x \quad 8)$

16.(12)



$y \quad mx \quad r\sqrt{a^2 m^2 \quad b^2}$

Equation of L_1 $k \quad mh \quad r\sqrt{16m^2 \quad 4}$

Equation of L_2 $y \quad \frac{1}{m}x$

$k \quad \frac{h}{m}$

$m \quad \frac{h}{k}$

$k \quad \left(\frac{h}{k}\right) \cdot h \quad r\sqrt{\frac{16h^2}{k^2} \quad 4}$

$\left(k \quad \frac{h^2}{k}\right)^2 \cdot \frac{16h^2 \quad 4k^2}{k^2}$

$\frac{(k^2 \quad h^2)^2}{k^2} \quad \frac{16h^2 \quad 4k^2}{k^2}$

$(k^2 \quad h^2)^2 \quad 16h^2 \quad 4k^2$

D 16, E 4

D E12

17.(B) Given curve is

$y^2 \quad 6x \qquad \ldots(i)$

Equation of normal to curve $y^2 \quad 4ax$ at point $(at^2, 2at)$ is given by $y \quad tx \quad 2at \quad at^3$ where

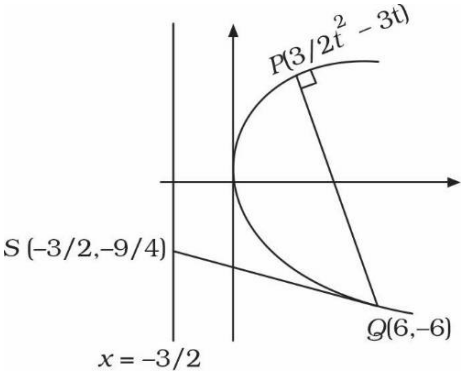
$a \quad \frac{3}{2}$

Now this normal is passing through point (5, –8)

$? \qquad 8 \quad 5t \quad 3t \quad \frac{3}{2}t^3$

$t \quad 2$ is a root of above equation

$? \qquad P(at^2, 2at) \quad P(6, 6)$



Equation of tangent at point $P \quad x \quad 2y \quad 6 \quad 0 \qquad \ldots(i)$

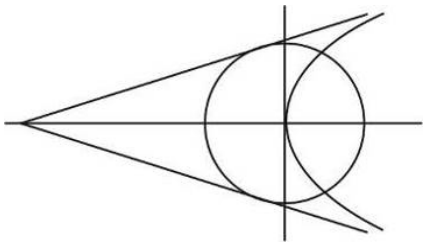
And equation of directrix is $x \quad \frac{3}{2} \qquad \ldots(ii)$

Now solve equation (i) and (ii)

$\ddot{Y} \qquad Q \quad \left(\frac{3}{2}, \frac{9}{4}\right)$

So required ordinate will be $\frac{9}{4}$

18.(4) The curves are $x^2 \quad y^2 \quad \frac{9}{4}$ and $y^2 \quad 4x$



Say the common tangent is $y \quad mx \quad \frac{1}{m}$

Since it's a tangent to the circle so

$$\left|\frac{1/m}{\sqrt{m^2 \quad 1}}\right| \quad \frac{3}{2}$$

$\ddot{Y} \qquad 4 \quad 9m^2(m^2 \quad 1) \qquad \ddot{Y} \qquad 9m^4 \quad 9m^2 \quad 4 \quad 0 \qquad \ddot{Y} \qquad (3m^2 \quad 4)(m^2 \quad 1/3) \quad 0$

So the point intersection in $Q \quad (\ 3, 0)$

$OQ \quad 3, a \quad 6$

$\frac{l}{e^2} \quad 4$

19.(D) Vertex (5, 4)

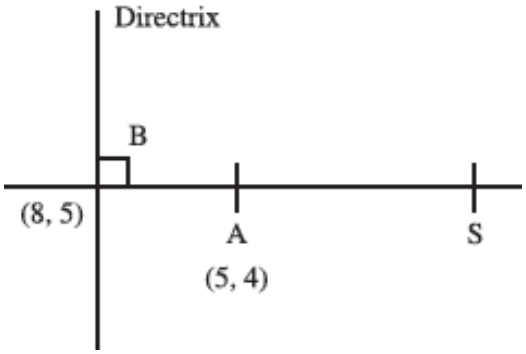
Directrix : $3x \quad y \quad 29 \quad 0$

Co-ordinates of B (foot of directrix)

$$\frac{x \quad 5}{3} \quad \frac{y \quad 4}{1} \quad \frac{3(5) \quad 4 \quad 29}{10} \quad 1$$

$x \quad 8, y \quad 5$

S $2,3$ (focus)



Equation of parabola

PS PM

So equation is

$x^2 \quad 9y^2 \quad 6xy \quad 134x \quad 2y \quad 711 \quad 0$

$a \quad b \quad c \quad d \quad k \quad 9 \quad 6 \quad 134 \quad 2 \quad 711 \quad 576$

20.(B) $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1 \qquad \ldots(1) \qquad a \neq b$

$? \qquad b^2 \quad a^2 \quad 1 \quad e^2 \quad e = \frac{1}{4}$

$b^2 \quad \frac{15a^2}{16} \qquad \ldots(2)$

As equation (1) passes through $\left(4\sqrt{\frac{2}{5}}, 3\right)$

So $\frac{32}{5a^2} \quad \frac{9}{b^2} \quad 1$

Replace b^2 from equation (2)

$\frac{32}{5a^2} \quad \frac{48}{5a^2} \quad 1 \quad Y \qquad 5a^2 \quad 80$

$\boxed{a^2 \quad 16}$ & $\boxed{b^2 \quad 15}$

So $\boxed{a^2 \quad b^2 \quad 31}$

21.(63) $r \quad 2; \quad V \quad 0,0 \; ; \quad S\left(\frac{1}{2}, 0\right)$

$x \quad h^2 \quad y \quad K^2 \quad 2^2$

$h^2 \quad k^2 \quad 4$

Solve $\left(h \quad \frac{1}{2}\right)^2 \quad k^2 \quad 4$

$h \quad \frac{1}{4}; \quad k^2 \quad \frac{63}{16}$

Solve circle & parabola

$$\left(x-\frac{1}{4}\right)^2+\left(x-\frac{1}{4}\right)^2+(x+k)^2=4$$

Let $\left(x-\frac{1}{4}\right)^2=t$

Equation will be of form

$At^2+Bt+C=0$

For one solution $D=0$

i.e. $\left(x-\frac{1}{4}\right)^2=\frac{-B\pm\sqrt{D}}{2A}$

Also $\frac{-B}{2A}\geq 0$ for one solution

$\therefore x\geq\frac{1}{4}$

$Y=(x+k)^2-4$

$(x+k)^2=Y+4$

$x+k=\sqrt{Y+4}$

$x=-k\pm\sqrt{Y+4}$, $-k-\sqrt{Y+4}$ not possible as $x\geq\frac{1}{4}$

$Y+4\geq\left(\frac{1}{4}+k\right)^2$

$Y+4\geq\left(\frac{1}{4}+k\right)^2$

22.(D) $e^2=\frac{11}{14}$

$\therefore 1+\frac{a^2}{b^2}=\frac{11}{14}\left(2+\frac{a^2}{b^2}\right)$

$\therefore 7+7a^2=b^2+11a^2b^2$ …(i)

$e'^2=\frac{11}{8}$

$\therefore 1+\frac{b^2}{a^2}=\frac{11}{8}\left(2+\frac{b^2}{a^2}\right)$

$8+8a^2=b^2+11a^2b^2$ …(ii)

$\frac{7}{4}=\frac{a}{b}\Rightarrow b=\frac{4}{7}a$

$\therefore 7+7a^2=b^2+11a^2b^2$

$7\left(1+\frac{b^2}{a^2}\right)=11b^2\Rightarrow b=\frac{65}{77}\Rightarrow a=\frac{7}{4}b=\frac{7}{4}\times\frac{65}{77}=\frac{65}{44}$.

Hence, $77a-44b=130$

23.(B) Clearly, $a = \frac{|8 \quad 3 \quad 21|}{5} = \frac{10}{5} = 2$

(Distance between vertex and directrix = a)

$\therefore 4a = 8$

24.(B) $e^2 = 1 + \frac{b^2}{a^2}$

$\frac{5}{2} = 1 + \frac{b^2}{a^2} \Rightarrow \frac{b^2}{a^2} = \frac{3}{2} \Rightarrow \frac{b}{a} = \sqrt{\frac{3}{2}}$

Length of latus rectum $= \frac{2b^2}{a} = 6\sqrt{2}$

$\Rightarrow \frac{2b^2}{a^2}, a = 6\sqrt{2}$

$3 \cdot \frac{2}{a} \cdot a = 6\sqrt{2}$

$a = 2\sqrt{2}$

$b = 2\sqrt{2} \cdot \sqrt{\frac{3}{2}} = 2\sqrt{3}$

So the hyperbola is $\frac{x^2}{8} - \frac{y^2}{12} = 1$

For the line $y = 2x + c$ to be tangent

$c^2 = 8(2)^2 + 12$

$c^2 = 32 + 12 = 20$

25.(D) Since the 2 tangents make angle of $\frac{S}{4}$ with the same line, the tangents are $\perp r$ and their points of contact are the extremities of a focal chord. Hence, A, B and S are always collinear irrespective of the value of a.

26.(B) Let $P(t^2, 2t)$

$Q \left(\frac{1}{t^2}, -\frac{2}{t}\right)$ {since PQ is a focal chord}

Let R = (3, 0)

Slope of $PR = \frac{2t}{t^2 - 3}$

Slope of $QR = \frac{-\frac{2}{t}}{\frac{1}{t^2} - 3} = \frac{2t}{3t^2 - 1}$

$PR \perp RQ \Rightarrow 4t^2 + (t^2 - 3)(3t^2 - 1) = 0$

$\Rightarrow t^2 = 1$

Ÿ P(1, 2) and Q(1, –2)

Since PQ is focal chord of ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ (a^2 > b^2)$

Focus of ellipse (1, 0) = (ae, 0)

Ÿ ae = 1

The given chord is perpendicular to the major axis of ellipse.

Ÿ $2 \frac{b^2}{a}$ {Ends of latus rectum are $\left(ae, \pm\frac{b^2}{a}\right)$}

Ÿ $b^2 = 2a$

Ÿ $a^2(1 - e^2) = 2a$

Ÿ $a^2 - 2a - 1 = 0$ Ÿ $a = \frac{1}{e} = \sqrt{2} + 1$

27.(88) Given $2a + 2b = 4\sqrt{2} + \sqrt{14}$

$a + b = 2\sqrt{2} + \sqrt{14}$ .....(1)

$e = \frac{\sqrt{11}}{2}$ Ÿ $e^2 = \frac{11}{4}$ Ÿ $e^2 = 1 + \frac{7}{4}$

Ÿ $\frac{b^2}{a^2} = \frac{7}{4}$ Ÿ $b = \frac{\sqrt{7}}{2}a$ .....(2)

From (1) & (2)

$\left(1 + \frac{\sqrt{7}}{2}\right)a = 2\sqrt{2} + \sqrt{7}$

Ÿ $a = 4\sqrt{2}$

$b = 2\sqrt{14}$

? $a^2 + b^2 = 32 + 56 = 88$

28.(A) $a = 7, b = 2\sqrt{6}$

$b^2 = a^2(1 - e^2)$

$e = \frac{5}{7}$

29.(D) $y^2 = 2x + 2y + 1$

$2y\frac{dy}{dx} = 2 + 2\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{y - 1}$

? $\left(\frac{dy}{dx}\right)_{1,3} = \frac{1}{2}$ and $\left(\frac{dy}{dx}\right)_{1,-1} = -\frac{1}{2}$

$y - 3 = \frac{1}{2}(x - 1)$

$2y - 6 = x - 1 \Rightarrow x - 2y + 5 = 0$ …(i)

$y - 1 = \frac{1}{2}(x - 1)$

$2y - 2 = x - 1 \Rightarrow x - 2y + 1 = 0$ …(ii)

$P(-3,1)$

Area of ΔPAB

$$\frac{1}{2}\begin{vmatrix} -3 & 1 & 1 \\ 1 & -3 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$\frac{1}{2}|-12 - 4| = 8$$

30.(D) $\frac{x^2}{16} + \frac{y^2}{7} = 1$ $\qquad e = \frac{3}{4}$

$\frac{x^2}{144} + \frac{y^2}{D} = \frac{1}{25}$

$\Rightarrow \frac{\frac{x^2}{144}}{\frac{1}{25}} + \frac{\frac{y^2}{D}}{\frac{1}{25}} = 1$

$4 \cdot \frac{3}{4} = \frac{12}{5} = e'$

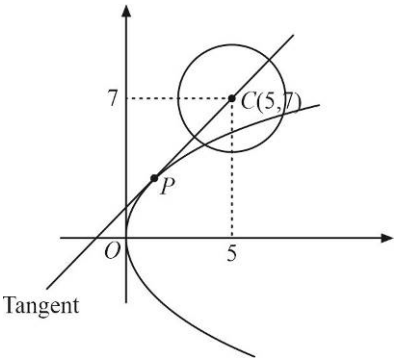
$e' = \frac{5}{4}$

$\frac{D}{25} = \frac{144}{25} \cdot \frac{9}{16} = 1$

$D = 81$

$\frac{2b^2}{a} = \frac{2 \cdot \frac{81}{25}}{\frac{12}{5}} = \frac{2 \cdot 81}{25} \cdot \frac{5}{12} = \frac{27}{10}$

31.(D)



Let point $P(2t^2, 2t) \equiv (2t^2, 4t)$ $\Rightarrow 4 = 8, t = 2$

Tangent at point

$ty = x + 2t^2$

If passes through C(5, 7)

$7t = 5 + 2t^2$

$2t^2 - 7t + 5 = 0$

$2t^2 - 5t - 2t + 5 = 0$

$t(2t - 5) - (2t - 5) = 0$

$(2t - 5)(t - 1) = 0$

$t = 1$ or $t = \frac{5}{2}$

A $2(1)^2 + 2\left(\frac{5}{2}\right)^2 = 4.\frac{25}{4} = 25$

B $16(1) + 8\left(\frac{5}{2}\right) = 40$

A + B = 65

32.(1552)

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

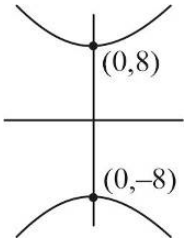
$\frac{x^2}{49} - \frac{y^2}{64} = -1$

$0 - \frac{y^2}{64} = -1$

$y^2 = 64$

$e_H = \frac{\sqrt{64 + 49}}{8}$

$e_H = \frac{\sqrt{113}}{8}$



$a^2 = b^2(1 - e^2)$

$e_E = \frac{\sqrt{a^2 + 64}}{8}$

$e_H = 2e_E = 1$

$\frac{\sqrt{113}}{8} = \frac{\sqrt{a^2 + 64}}{8} = \frac{1}{2}$

113 a^2 64 1024

a^2 64 $\frac{1024}{113}$

l $\frac{2a^2}{b}$ $\frac{2\left(\frac{1024}{113}\right) 64}{8}$ $\frac{231}{113}$ 16 p

l $\frac{2039}{113}$

113l 2039

33.(75) Given ellipse is

x^2 2x 4y 8y O 5

1 4

Ÿ (x 1)^2 4(y 2)^2 O 5

$\frac{(x\ 1)^2}{O\ 5}$ $\frac{(y\ 2)}{\left(\frac{O\ 5}{4}\right)}$ 1 Ÿ $\frac{x^2}{a^2}$ $\frac{y^2}{b^2}$ 1

Here a^2 O 5 and b^2 $\frac{O\ 5}{4}$

Length of latus – rectum $\frac{2b^2}{|a|}$

$2\left(\frac{O\ 5}{4}\right)$ $\frac{1}{\sqrt{O\ 5}}$ 4

Ÿ $\sqrt{O\ 5}$ 2 4 Ÿ O 5 64

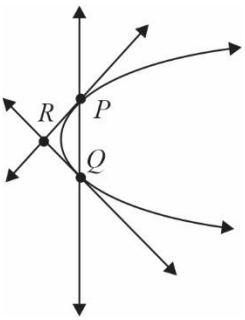
? O 64 5 59

O 59

Length of major a u13 2a 2 $\sqrt{O\ 5}$ 2 $\sqrt{59\ 5}$ 2 8 u 16

So, O l 59 16 75

34.(B) R 0,1



Parabola : y^2 2x 3 ….(i)

Chord of contact : S_1 T

i.e. yy_1 x x_1 3

Ÿ y 1 x 0 3

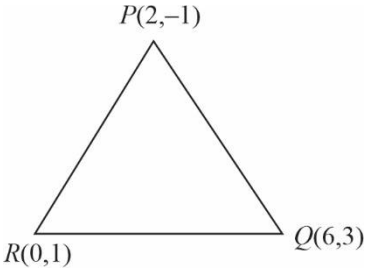
Ÿ y x 3 ….(ii)

(i) & (ii) Ÿ x 2 or 6 Ÿ y 1 or 3

So, vertices of 'PQR are 0,1 , 2, 1 and 6,3

Here $m_{PR}\ m_{PQ}$ 1 Ÿ PR APQ

Ÿ 2, 1 is the orthocenter.



35.(3) $H: \frac{x^2}{1} \frac{y^2}{1} 1$ & $E: \frac{x^2}{a^2} \frac{y^2}{b^2} 1;\ a\ !b\ !0$

$e_E \sqrt{1 \frac{b^2}{a^2}} \frac{1}{e_H} \frac{1}{\sqrt{1 \frac{1}{1}}} \frac{1}{\sqrt{2}}$

Ÿ $\frac{a^2 b^2}{a^2} \frac{1}{2}$ Ÿ $a^2 2b^2$ …..(i)

$y \sqrt{\frac{5}{2}}x$ K be the common tangent

K $r\sqrt{a^2m^2 b^2}$ $r\sqrt{a^2 \left(\sqrt{\frac{5}{2}}\right)^2 b^2}$ {for ellipse}

& k $r\sqrt{a_1^2m^2 b_1^2}$ $r\sqrt{a_1^2 \left(\sqrt{\frac{5}{2}}\right)^2 b_1^2}$ $r\sqrt{\frac{5}{2} 1}$ $r\sqrt{\frac{3}{2}}$

Equating Ÿ $a^2 \left(\sqrt{\frac{5}{2}}\right)^2 b^2$ $3/2$ Ÿ b^2 $\frac{1}{4}$ {using (i)} & a^2 $\frac{1}{2}$

? 4 a^2 b^2 3

36.(2) 2p f 1 0 …(i)

2 pf 4p 0 …(ii)

2 p(f 4)

$p \frac{2}{f 4}$

2p 1 f

$\frac{4}{f 4}$ 1 f

$f^2 \quad 3f \quad 0$

$f \quad 0 \text{ or } \quad 3$

Hyperbola $3x^2 \quad y^2 \quad 3, \; x^2 \quad \frac{y^2}{3} \quad 1$

$y \quad mx \quad r\sqrt{m^2 \quad 3}$

It passes (1,0)

$0 \quad m \quad r\sqrt{m^2 \quad 3}$

m tends f

it passes (1,3)

$3 \quad m \quad r\sqrt{m^2 \quad 3}$

$(3 \quad m)^2 \quad m^2 \quad 3$

$m \quad 2$

37.(10) $x^2 \quad \frac{64.5}{75}\left(y \quad \frac{3}{5}\right)$

Equation of tangent at $\left(\frac{8}{5}, \frac{6}{5}\right)$

$x\left(\frac{8}{5}\right) \quad \frac{64}{15}\left(\frac{y \quad \frac{6}{5}}{2}\right) \quad \frac{3}{5}$

$3x \quad 4y \quad 0$

Equation of family of circle is

$\left(x \quad \frac{8}{5}\right)^2 \quad \left(y \quad \frac{6}{5}\right)^2 \quad (3x \quad 4y) \quad 0$

It touches y axis so $f^2 \quad c$

$x^2 \quad y^2 \quad x\left(3 \; O \; \frac{16}{5}\right) \quad y\left(4 \; O \; \frac{12}{5}\right) \quad 4 \quad 0$

$\frac{\left(4 \; O \; \frac{12}{5}\right)^2}{4} \quad 4$

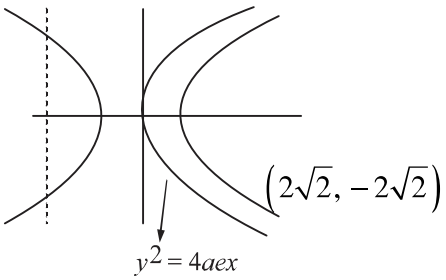
$O \; \frac{2}{5} \text{ or } \quad O \; \frac{8}{5}$

$O \; \frac{2}{5}, r \quad 1$

$O \quad \frac{8}{5}, r \quad 4$

$d_1 \quad d_2 \quad 10$

38.(B)

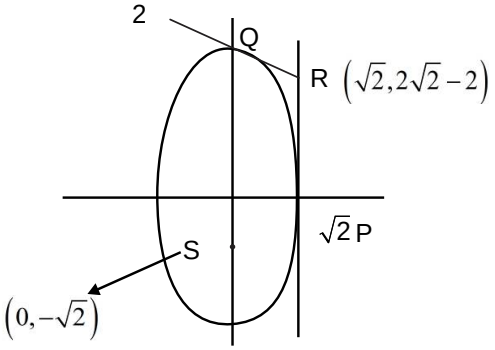


$\frac{8}{a^2} \quad \frac{8}{b^2} \quad 1$

$4ae \quad e \; u\frac{2b^2}{a} \quad Ÿ2a^2 \quad b^2$

$Ÿ \; a^2 \quad 4, b^2 \quad 8$

39.(13)



Solving c.o.c with ellipse gives $P(\sqrt{2}, 0)$, $Q(1, \sqrt{2})$

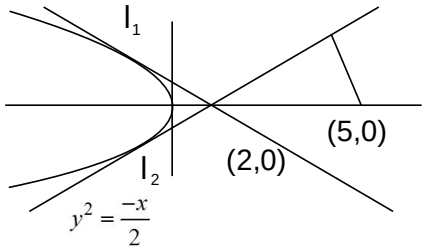
$SP^2 \quad SQ^2 \quad 4 \quad 9 \quad 13$

40.(9) $y \quad mx \quad \frac{1}{8m}$

$m \quad r\frac{}{4}$

$y \quad \frac{1}{4}x \quad \frac{1}{2}$ or $x \quad 4y \quad 2 \quad 0$

$\sqrt{r} \quad \frac{3}{\sqrt{17}}$



41.(B) Line is passing through intersection of

$bx \quad 10y \quad 8 \quad 0$ and $2x \quad 3y \quad 0$ is $bx \quad 10y \quad 8 \quad O2x \quad 3y \quad 0$.

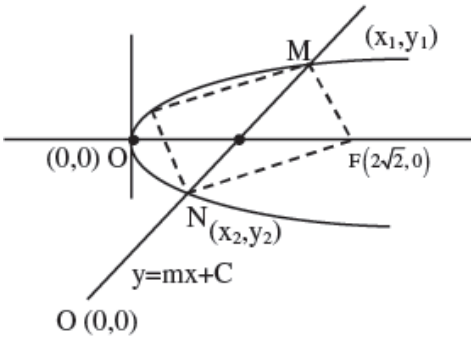
As line is passing through (1, 1) so $O \; b \quad 2$

Now line $3b \quad 4 \; x \quad 3b \quad 4 \; y \quad 8 \quad 0$ is tangent to circle $17 \; x^2 \quad y^2 \quad 16$

So $\frac{8}{\sqrt{3b \quad 4^{\;2} \quad 3b \quad 4^{\;2}}} \quad \frac{4}{\sqrt{17}}$

$Ÿ \quad b^2 \quad 2 \; Ÿ e \quad \sqrt{\frac{3}{5}}$

42.(B)



H : $\frac{x^2}{4} \quad \frac{y^2}{4} \quad 1$

Focus (ae, 0)

F $2\sqrt{2},0$

Line L : y mx c pass (1, 0)

o m C ….(1)

Line L is tangent to Hyperbola $\frac{x^2}{4} \quad \frac{y^2}{4} \quad 1$

C $r\sqrt{a^2m^2 \quad ^2}$

C $r\sqrt{4m^2 \quad 4}$

From (1)

m $r\sqrt{4m^2 \quad 4}$

Squaring

$m^2 \quad 4m^2 \quad 4$

$4 \quad 3m^2$

$\frac{2}{\sqrt{3}}$ m (as m > 0)

C m

C $\frac{2}{\sqrt{3}}$

y $\frac{2x}{\sqrt{3}} \quad \frac{2}{\sqrt{3}}$

$y^2 \quad 4x$

Ÿ $\left(\frac{2x \quad 2}{\sqrt{3}}\right)^2 \quad 4x$

Ÿ $x^2 \quad 1 \quad 2x \quad 3x$

Ÿ $x^2 \quad 5x \quad 1 \quad 0$

$y^2 \quad 4\left(\frac{\sqrt{3}y \quad 2}{2}\right)$

$y^2 \quad 2\sqrt{3}y \quad 4$

$\ddot{Y} \quad y^2 \quad 2\sqrt{3}y \quad 4 \quad 0$

Area

$$\left|\frac{1}{2}\begin{vmatrix} 0 & x_1 & 2\sqrt{2} & x_2 & 0 \\ 0 & y_1 & 0 & y_2 & 0 \end{vmatrix}\right|$$

$$\left|\frac{1}{2}\begin{vmatrix} a & & & & \\ \neg & 2\sqrt{2}y_1 & 2\sqrt{2}y_2 & & c \end{vmatrix}\right|$$

$$\sqrt{2}\,|y_2 \quad y_1| \quad \frac{\sqrt{2}\ \sqrt{12 \quad 16}}{111}$$

$\sqrt{56}$

$2\sqrt{14}$

43.(B) Equation of pair of tangents

$SS_1 \quad T^2$

$(2x^2 \quad 3y^2 \quad 5)\,u24 \quad 2x \quad 9y \quad 5\ ^2$

$44x^2 \quad 9y^2 \quad 36xy \quad 20x \quad 90y \quad 145 \quad 0$

$$\tan T \ \frac{2\sqrt{h^2 \quad ab}}{a \quad b} \quad \frac{24}{7\sqrt{5}} \ \ddot{Y}\ T \tan \quad 1 \ \frac{\S 24}{\cdot\cdot 7\sqrt{5}}$$

44.(B) $y \quad x^2,\ T : y \quad mx \quad \frac{1}{4}m^2$

$y \quad (x \quad 2)^2,\ T : y \quad m(x \quad 2) \quad \frac{1}{4}m^2$

Now, $2m \quad \frac{1}{4}m^2 \quad \frac{1}{4}m^2$

$m \quad 0, 4$

$T : y \quad 4(x \quad 1)$

45.(C) $\frac{x^2}{6/k} \quad \frac{y^2}{6} \quad 1$

Direcrtix : $x \quad 1 \quad \frac{a}{3}\ \ddot{Y}\ \boxed{a \quad e}$

Now, $b^2 \quad a^2(e^2 \quad 1)$

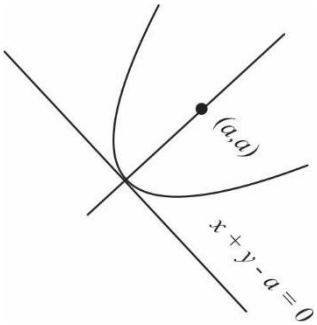
$$6 \quad \frac{6}{k} \ \frac{\S 6}{\cdot\cdot k} \quad 1 \ \ddot{Y}\ k \quad 2 \text{ or } k \quad 3$$

but $k > 3$, so $\boxed{k \quad 2}$

Hyperbola is $2x^2 \quad y^2 \quad 6$

$\sqrt{5}, \quad 2$ satisfies.

46. (C)



l 4A

l 4 u $\frac{a}{\sqrt{2}}$ 16 Ÿa 4 $\sqrt{2}$

47.(20)

y mx r $\sqrt{a^2 m^2 b^2}$

$4m^2$ 9 $42m^2$ 143

$38m^2$ 52

m r2 !0

m 2

y 2x r5

2x y 5 0

Tangent for ellipse

$\frac{xx_1}{4}$ $\frac{yy_1}{9}$ 1

Tangent for hyperbola

$\frac{xx_2}{42}$ $\frac{yy_2}{143}$ 1

On comparing with 2x y 5

We get x_1 $\frac{8}{5}, x_2$ $\frac{84}{5}$

Ÿ $|2x_1 x_2|$ 20

48.(D) Tangent at D, E has slope 1

E^2 24 [

Equation of tangent y E 12 x D, $\frac{12}{E}$ 1

Ÿ D 6, E 12

? D 4, E 4 10, 16

Normal at (10, 16) to $\frac{x^2}{36}$ $\frac{y^2}{144}$ 1 is

2x 5y 100

49.(45) f cx 4x $\frac{1}{x}$

a $\frac{1}{2}$

Let P x_1, y_1 be any point on y^2 4ax

$\frac{1}{y_1}\frac{3}{4}\frac{y_1}{x_1}$ Ÿ y_1^2 $6y_1$ 8 0

y_1 2, 4 Ÿ P(8, 4) as P(2, 2) rejected

Equation of normal at P.

y 4 4 x 8

$\frac{x}{9}$ $\frac{y}{36}$ 1

D 9, E 36

D E 45

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Inverse Trigonometric Functions	Class- XII | Mathematics

JEE Main 2021

1.(1) $\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}(r+1)-\tan^{-1}r\right)$, $\tan\left(\lim_{n\to\infty}\tan^{-1}\frac{1}{1+n(n+1)}\right)$, $\tan\left(\frac{\pi}{4}-0\right)$, $\tan\frac{\pi}{4}=1$

2.(A) Let $2\cot^{-1}5+\cos^{-1}\frac{4}{5}=\cot\frac{\theta}{2}$

$\therefore \tan\frac{\theta}{2}=\frac{1}{5}$? $\tan\theta=\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}=\frac{\frac{2}{5}}{1-\frac{1}{25}}=\frac{2}{5}\times\frac{25}{24}$

$\tan\theta=\frac{5}{12}$ $\therefore \theta=\tan^{-1}\frac{5}{12}$? $\operatorname{cosec}\left(\tan^{-1}\frac{5}{12}+\cos^{-1}\frac{4}{5}\right)$

$=\operatorname{cosec}\left(\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{3}{5}\right)$ $=\operatorname{cosec}\left(\sin^{-1}\frac{5}{13}\times\frac{4}{5}+\frac{12}{13}\times\frac{3}{5}\right)$

$=\operatorname{cosec}\left(\sin^{-1}\frac{56}{65}\right)$ $=\operatorname{cosec}\left(\cos^{-1}\frac{33}{65}\right)=\frac{65}{56}$

3.(A) $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c}=\lambda$

$\sin^{-1}x+\cos^{-1}x=a+b=\lambda$

$\frac{\pi}{2}=a+b+\lambda$ ……..(i)

$\tan^{-1}y=c\lambda$ ……..(ii)

$\frac{\pi}{2\tan^{-1}y}=\frac{a+b}{c}$

$\frac{\pi}{a+b}=2\tan^{-1}y$

$\cos\left(\frac{\pi}{a+b}\right)=\cos 2\tan^{-1}y=\frac{1-y^2}{1+y^2}$

4.(D) $0<a,b<1$ & $\tan^{-1}a+\tan^{-1}b=\pi/4$

$\therefore \tan^{-1}\left(\frac{a+b}{1-ab}\right)=\pi/4$ $(\because ab<1)$

$\therefore \frac{a+b}{1-ab}=\tan\frac{\pi}{4}=1\Rightarrow a+b=ab+1$ ….(i)

Let $S=(a+b)+\left(\frac{a^2+b^2}{2}\right)+\left(\frac{a^3+b^3}{3}\right)+\left(\frac{a^4+b^4}{4}\right)+\ldots$

$=\left(a+\frac{a^2}{2}+\frac{a^3}{3}+\frac{a^4}{4}+\ldots\right)+\left(b+\frac{b^2}{2}+\frac{b^3}{3}+\frac{b^4}{4}+\ldots\right)$

$=\ln(1+a)+\ln(1+b)=\ln(1+a+b+ab)=\ln(1+1)$ (using (1))

$=\ln 2$

5.(C) Let $\sin^{-1}\frac{\sqrt{63}}{8} = \theta \quad \sin\theta = \frac{\sqrt{63}}{8} \quad \cos\theta = \frac{1}{8}$

$$\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{1}{8}$$

Apply C & D $\quad \tan^2\frac{\theta}{2} = \frac{7}{9}$

$$\tan\frac{\theta}{2} = \frac{\sqrt{7}}{3}$$

$$\cos\frac{\theta}{2} = \frac{3}{4}$$

$$\frac{1-\tan^2\frac{\theta}{4}}{1+\tan^2\frac{\theta}{4}} = \frac{3}{4}$$

Apply C & D $\quad \tan^2\frac{\theta}{4} = \frac{1}{7}$; $\tan\frac{\theta}{4} = \frac{1}{\sqrt{7}}$

6.(B) $\sin^{-1}\left(x^2 - \frac{1}{3}\right) + \cos^{-1}\left(x^2 - \frac{2}{3}\right) = x^2$

$x \in [-1,1]$

$\sin^{-1}\left(x^2 - \frac{2}{3}\right) + \frac{\pi}{2} + \cos^{-1}\left(x^2 - \frac{2}{3}\right) = x^2$

$0 \le x^2 \le 1$

$-\frac{2}{3} \le x^2 - \frac{2}{3} \le \frac{1}{3}$

$\therefore \left(x^2 - \frac{2}{3}\right) = 0 \quad \ldots(i)$

For both cases $x^2 \neq 0$
Which doesn't satisfy (i) Hence no solution

7.(B) $f(x) = \frac{\operatorname{cosec}^{-1}x}{\sqrt{x-[x]}}$

$x \in (-\infty,-1] \cup (1,\infty) - I$

8.(B) $\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}} + \frac{4x}{5}\sqrt{1-\frac{9x^2}{25}} = x$; $x = 0$ or $3\sqrt{25-16x^2} + 4\sqrt{25-9x^2} = 25$

$3\sqrt{25-16x^2} = 25 - 4\sqrt{25-9x^2}$

$9(25-16x^2) = 625 + 16(25-9x^2) - 200\sqrt{25-9x^2} \Rightarrow 200\sqrt{25-9x^2} = 400 - 25x^2 \Rightarrow \sqrt{25-9x^2} = 4 - x^2$

Finally $x \in \{-1,0,1\}$

9.(D) $\cot^{-1}\theta = \sum_{r=1}^{100}\tan^{-1}\left(\frac{1}{2r^2}\right) = \sum_{r=1}^{100}\tan^{-1}\left(\frac{2}{1+4r^2-1}\right) = \sum_{r=1}^{100}\tan^{-1}2r - \tan^{-1}2r-1$

$\tan^{-1}201 - \tan^{-1}1 = \tan^{-1}\left(\frac{200}{1+201}\right) = \tan^{-1}\left(\frac{200}{202}\right) \Rightarrow \cot^{-1}\left(\frac{101}{100}\right) = \theta = 1.01$

10.(B) $\tan^{-1}x - 1 + \tan^{-1}x - 1 = \tan^{-1}\left(\frac{8}{31}\right) \quad x - 1 \neq 0 \Rightarrow \boxed{x \neq 1}$

$$\tan^{-1}\left(\frac{2x}{1-x^2-1}\right) = \tan^{-1}\left(\frac{8}{31}\right)$$

$$\frac{2x}{2-x^2} = \frac{8}{31}$$

$31x \quad 8 \quad 4x^2$

$4x^2 \quad 31x \quad 8 \quad 0$

$x \quad 8$ and $x \quad \frac{1}{4}$

But $x \quad \frac{1}{4}$ doesn't salsify equation § ' LHS ! $\frac{S}{4}$ and RHS $\frac{S}{4}$

? Only $x \quad 8$ is correct.

11 .(A) ' $S_k \quad \sum_{r \quad 1}^{k} \tan^{1} \frac{2^r . 3^r}{2^{2r \quad 1} \quad 1 \quad \frac{3}{2}^{2r \quad 1}}$ Ÿ $S_k \quad \sum_{r \quad 1}^{k} \tan^{1} \frac{\frac{3^r}{2^{r \quad 1}}}{1 \quad \frac{3}{2}^{r \quad 1} \quad \frac{3}{2}^{r}}$

Ÿ $S_k \quad \sum_{k \quad 1}^{k} \tan^{1} \frac{\frac{3}{2}^{r \quad 1} \quad \frac{3}{2}^{r}}{1 \quad \frac{3}{2}^{r \quad 1} \quad \frac{3}{2}^{r}}$ Ÿ $S_k \quad \tan^{1} \frac{3}{2}^{k \quad 1} \quad \tan^{1} \frac{3}{2}$

? $\lim_{k \text{ o f}} S_k \quad \frac{S}{2} \quad \tan^{1} \frac{3}{2} \quad \cot^{1} \frac{3}{2}$

12.(C) ' $\tan^{1} \sqrt{x^2 \quad x} \quad \sin^{1} \sqrt{x^2 \quad x \quad 1} \quad \frac{S}{4}$

For equation to be defined, $x^2 \quad x \text{ t } 0$ Ÿ $x^2 \quad x \quad 1 \text{ t } 1$

? only possibility that equation is defined is

$x^2 \quad x \quad 1 \quad 1$ Y $x^2 \quad x \quad 0$ Y $x \quad 0, \ 1$

And none of these satisfy the given equation ? Number of roots = 0

13 .(B) $\tan \tan^{1} \frac{2 \text{ u} \frac{3}{5}}{1 \quad \frac{9}{25}} \quad \tan^{1} \frac{5}{12}$

$\tan \tan^{1} \frac{6 \text{ u} 5}{16} \quad \tan^{1} \frac{5}{12}$ $\tan \tan^{1} \frac{15}{6} \quad \tan^{1} \frac{5}{12}$

$\tan \tan^{1} \frac{\frac{15}{b} \quad \frac{5}{12}}{1 \quad \frac{15}{8} \text{ u} \frac{5}{12}}$ $\frac{15 \text{ u} 12 \quad 40}{8 \text{ u} 12 \quad 15 \text{ u} 5} \quad \frac{180 \quad 40}{96 \quad 75} \quad \frac{220}{21}$

14 .(D) $0 \quad dx^2 \quad x \quad 1 \quad d1$ & $0 \quad \frac{2x \quad 1}{2} \quad 1d$

Ÿ $x^2 \quad x \quad 1 \text{ t } 0$ & $x^2 \quad x \quad d0$ & $0 \quad 2 \text{ x } 1 \quad 2d$

Ÿ $x \bullet R$ & $x \bullet [0,1]$ & $\frac{1}{2} \quad x \quad d\frac{3}{2}$ Ÿ $x \bullet (\frac{1}{2},1]$ Ÿ D $1/2$ & E 1 Ÿ D E 3/2

15 .(A) I $1 \quad d\frac{x \quad 1}{x \quad 1} \quad d1$

$\frac{x \quad 1}{x \quad 1} \quad 1 \quad d0$

$\frac{x \quad 1 \quad x \quad 1}{x \quad 1} \quad d0$

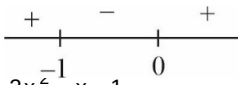
$\frac{2}{x \quad 1} \quad d0$

$\frac{2}{x \quad 1} \text{ t } 0$

$x \ ! \ 1$

II $1\ d\frac{x\ 1}{x\ 1}$ Ÿ $\frac{x\ 1\ x\ 1}{x\ 1}$ t0

$\frac{2x}{x\ 1}$ t 0



x • f, 1 ‰0, f

III $\frac{3x^2\ x\ 1}{x\ 1^2}$ d1

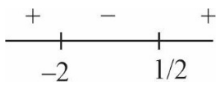
$3x^2\ x\ 1\ dx^2\ 2x\ 1$

$2x^2\ 3x\ 2\ d0$

$2x^2\ 4x\ x\ 2\ d0$

$2x\ x\ 2\ x\ 2\ d0$

$x\ 2\ 2x\ 1\ d0$



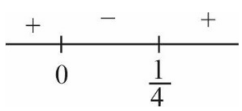
x • a 2, $\frac{1}{2}$ ‘

IV $\frac{3x^2\ x\ 1}{x\ 1^2}$ t 1

Ÿ $3x^2\ x\ y\ t\ x^2\ 2x\ 1$

$4x^2\ x\ t0$

$x\ 4x\ 1\ t0$



x • (f ,0] ‰ a $\frac{1}{4}$, f) 1 ` By all the conditions x • a $\frac{1}{4}$, $\frac{1}{2}$ o ‰ ‘0 `

16 .(C) ' a sin $^1x^2$ cos $^1x^2$

Ÿ a sin ^1x cos ^1x sin ^1x cos ^1x

Ÿ a $\frac{S}{2}$ § $\frac{S}{2}$ 2cos ^1x Ÿ 2cos ^1x $\frac{S}{2}$ $\frac{2a}{S}$ Ÿ $2x^2$ 1 cos ^{1} § $\frac{S}{2}$ $\frac{2a}{S}$ ¸ sin $\frac{2a}{S}$

17 .(B) $\sum_{r=1}^{50}$ tan $^1\frac{1}{2r^2}$ $\sum_{r=1}^{50}$ tan ^{1} § $\frac{(2r\ 1)\ (2r\ 1)}{1\ (2r\ 1)(2r\ 1)}$ $\sum_{r=1}^{50}$ tan $^1(2r\ 1)$ tan $^1(2r\ 1)$

tan ^{1101} tan ^{1} 1 tan $^1\frac{101\ 1}{1\ 101\ 1}$ tan ^{1} § $\frac{100}{102}$ ¸ p Ÿ tan p $\frac{100}{102}$ $\frac{50}{51}$

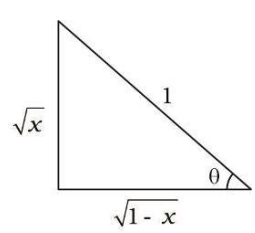
18 .(A) f x cos § 2tan ^{1} § sin § cot $^1\sqrt{\frac{1\ x}{x}}$ ¸ ¸ 0 x 1

1 x ! 0 & x ! 0 ' x • 0,1 `

Hence, $\sqrt{\frac{1\ x}{x}}$ $\frac{\sqrt{1\ x}}{\sqrt{x}}$! 0

? f x cos 2tan ^{1} sin T

{where cot $^1\sqrt{\frac{1\ x}{x}}$ T Ÿ sin ^{1} § $\frac{\sqrt{x}}{1}$ ¸ T Ÿ sin T $\sqrt{x}$ }



Ÿ f x cos 2tan $^{1}\sqrt{x}$ cos 2 I {where I tan $^{1}\sqrt{x}$ Ÿ tan I $\frac{\sqrt{x}}{1}$ }

Ÿ f x $\frac{1 \tan^{2} I}{1 \tan^{2} I} \frac{1 x}{1 x}$

f ' x $\frac{2}{1 x^{2}}$ Ÿ 1 x ^{2} f ' x 2 f x ^{2} 2 $\left(\frac{1 x}{1 x}\right)^{2}$ 2 $\frac{1}{1 x}$ 0

19.(A) cos ^{1}(cos(5)) 2 S 5 ...(i)

sin ^{1}(sin 6) 6 2 S ...(ii)

tan ^{1}(tan12) 4 S12 ...(iii)

(i) + (ii) – (iii) 4 S 11

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1.(AB) (A) $S_n = \sum_{k=1}^{n} \tan^{-1}\left(\frac{x}{1 + (k-1)x + kx^2}\right)$

$S_n = \sum_{k=1}^{n} \tan^{-1}((k+1)x) - (\tan^{-1}kx)$

$S_n = \tan^{-1}(n+1)x - \tan^{-1}(x)$

$S_{10} = \tan^{-1}11x - \tan^{-1}(x)$

$\tan^{-1}\left(\frac{10x}{1 + 11x^2}\right) = \frac{S}{2} \cot^{-1}\left(\frac{10x}{1 + 11x^2}\right) = \frac{S}{2} \tan^{-1}\left(\frac{1 + 11x^2}{10x}\right)$

(B) $S_{f(x)} = \frac{S}{2} \tan^{-1}(x)$ x !0 ; Hence B is correct

(C) $\tan^{-1}\left(\frac{3x}{1 + 4x^2}\right) = \frac{S}{4}$

1 4x ^{2} 3x No solution

(D) Counter example x $\frac{1}{2}$,n 2

$\tan\left(\tan^{-1}\left(\frac{2x}{1 + 3x^2}\right)\right) = \frac{1}{2}$

$\frac{1}{1 \frac{3}{4}} = \frac{1}{2}$; $\frac{4}{7} = \frac{1}{2}$ (False)

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1.(B) $\tan\left(2\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{8}\right)=\tan\left(\tan^{-1}\frac{1}{2}\right)$

$\tan\left(\tan^{-1}\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{40}}\right)=\tan\left(\tan^{-1}\frac{1}{2}\right)$

$\Rightarrow \tan\left(\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}\right)$ $\Rightarrow \tan\left(\tan^{-1}\frac{\frac{2}{3}+\frac{1}{9}}{1-\frac{1}{9}}\right)=\tan\left(\tan^{-1}\frac{1}{2}\right)$

$\Rightarrow \tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{2}\right)$ $\Rightarrow \tan\left(\tan^{-1}\frac{\frac{3}{4}+\frac{1}{2}}{1-\frac{3}{4}\cdot\frac{1}{2}}\right)$ $\Rightarrow \tan\left(\tan^{-1}2\right)=2$

2.(B) $\frac{\sin^{-1}x}{D}+\frac{\cos^{-1}x}{E}=\frac{\sin^{-1}x\cos^{-1}x}{DE}=\frac{S}{2(DE)}$

Now $\frac{\sin^{-1}x}{D}=\frac{S}{2(DE)}\Rightarrow x=\sin\left(\frac{SD}{2(DE)}\right)=\sin T$

$\sin\left(\frac{2SD}{DE}\right)=\sin 4T=2\sin 2\theta\cos 2\theta$

$2\cdot 2x\sqrt{1-x^2}(1-2x^2)$

$4x\sqrt{1-x^2}(1-2x^2)$

3.(A) $f'(x)=\frac{1}{1+4x^2}>0$ at $t\ge 0$

and $-\frac{\pi}{6}\le x\le\frac{\pi}{6}$

$0\le x^2\le\frac{\pi^2}{36}$

$\Rightarrow \frac{18}{\pi^2+9}\le a\le\frac{2}{1+4x^2}\le a\le 2$ $\Rightarrow \frac{18}{\pi^2+9}\le a\le 2$ $\Rightarrow a\in\left[\frac{6}{\pi^2+9},2\right]$

$\text{Max}(\bar{a})=\frac{6}{\pi^2+9}$

$f(\bar{a})=\tan^{-1}(2x)+3\bar{a}x+7\Big|_{\text{at } x=\frac{\pi}{8}}$ $\Rightarrow \tan^{-1}\left(\frac{\pi}{4}\right)+7\cdot\frac{3\pi}{8}\cdot\frac{6}{\pi^2+9}$

So, no option is correct

(Actually, NTA has assumed $\tan^{-1}\left(\frac{\pi}{4}\right)$ as 1 which is wrong)

4.(B) $(x * 1) * 1 = x * (1 * 1)$

$(x^2 - 1) * 1 = x * 2$

$(x^2 - 1)^2 - 1 = x^2 - 8$

$x^4 - x^2 - 6 = 0 \Rightarrow (x^2 - 3)(x^2 + 2) = 0$

Thus, $x^2 = 3$ since $x^2 \geq 3$

$\therefore \sin^{-1}\left(\frac{x^4 - x^2 + 2}{x^4 + x^2 + 2}\right)$

Putting $x^2 = 2$

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} = \frac{\pi}{3}$

5.(B) $y = \tan^{-1}\left(\frac{1 - \sin x^3}{\cos x^3}\right) = \tan^{-1}\left(\frac{\cos\frac{\pi}{2} - x^3}{\sin\frac{\pi}{2} - x^3}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x^3}{2}\right)\right)$

Since $\frac{\pi}{4} - \frac{x^3}{2} \in \left(0, \frac{\pi}{2}\right) \Rightarrow y = \frac{\pi}{4} - \frac{x^3}{2}$

$\therefore y' = -\frac{3x^2}{2}$

$y' = -3x$

$\therefore x^2 y'' + 6y = \frac{3\pi}{2}$

6.(A) $(\tan^{-1}x)^3 + (\cot^{-1}x)^3 = kx^3$ let $\tan^{-1}x = t$

$y = t^3 + \left(\frac{\pi}{2} - t\right)^3 \Rightarrow y = t^3 + \frac{\pi^3}{8} - t^3 - \frac{3}{2}t^2\pi + \frac{3}{4}t\pi^2$

$y = \frac{3\pi}{2}t^2 - \frac{3\pi^2}{4}t + \frac{\pi^3}{8}$ $t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\frac{3\pi}{2}t^2 - \frac{\pi^2}{2}t + \frac{\pi^3}{16} \neq \frac{3x^3}{32} + \frac{\pi^3}{8}$

$\frac{3\pi}{2}t^2 - \frac{\pi^2}{4}t + \frac{\pi^3}{32} \neq \frac{3x^3}{32}$

$t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow t \in \left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$

$\therefore t^2 \in \left[0, \frac{9\pi^2}{16}\right)$

$\therefore (\tan^{-1}x)^3 + (\cot^{-1}x)^3 \in \left[\frac{\pi^3}{32}, \frac{7\pi^3}{8}\right)$

$\therefore k \in \left[\frac{1}{32}, \frac{7}{8}\right)$

7.(B) $\tan^{-1}\left(\frac{\frac{1}{\sqrt{2}} - 1}{\frac{1}{\sqrt{2}} + 1}\right) = \tan^{-1}1 - \tan^{-1}\sqrt{2} = \tan^{-1}\sqrt{2} - 1 = \frac{\pi}{8}$

8.(C) $\cos^{-1}\frac{3}{10}$ $\cos\tan^{-1}\frac{4}{3}$ $\frac{2}{5}$ $\sin\tan^{-1}\frac{4}{3}$

$\cos\tan^{-1}\frac{4}{3}$ $\frac{3}{5}$ and $\sin\tan^{-1}\frac{4}{3}$ $\frac{4}{5}$

$\cos^{-1}\frac{9}{50}$ $\frac{8}{25}$ $\cos^{-1}\frac{1}{2}$ $\frac{S}{3}$

9.(B) $f(x)$ $2\cos^{-1}x$ $4\cot^{-1}x$ $3x^2$ $2x$ 10

Notice f is decreasing and continuou s in [–1, 1]

f_{max} $f(1)$ $2S$ 3 $S9$ 5 $S9$ b

f_{min} $f(1)$ S 5 a

Range [S 5, 5 S9]

$4a$ b 11 S

10.(A) $\tan^{-1}\frac{1}{1\ n\ n^2}$ $\tan^{-1}\frac{n\ 1\ n}{1\ n\ n\ 1}$

$\tan^{-1}n\ 1$ $\tan^{-1}n$

So, $\sum_{n=1}^{50}\tan^{-1}n\ 1$ $\tan^{-1}n$

$\tan^{-1}51$ $\tan^{-1}1$

$\cot\sum_{n=1}^{50}\tan^{-1}\frac{1}{1\ n\ 1^2}$ $\cot\tan^{-1}51$ $\tan^{-1}1$

$\frac{1}{\tan\tan^{-1}51\ \tan^{-1}1}$ $\frac{1\ 51\ 1}{51\ 1}$ $\frac{52}{50}$ $\frac{26}{25}$

11.(A) $\sin^{-1}\sin\frac{2S}{3}$ $\cos^{-1}\cos\frac{7S}{6}$ $\tan^{-1}\tan\frac{3}{4}$ $\sin^{-1}\sin\frac{S}{3}$ $\cos^{-1}\cos\frac{S}{6}$ $\tan^{-1}\tan\frac{S}{9}$

$\sin^{-1}\sin\frac{S}{3}$ $\cos^{-1}\cos\frac{S}{6}$ $\tan^{-1}\tan\frac{S}{4}$ $\frac{S}{3}$ S $\frac{S}{6}$ $\frac{S}{4}$ $\frac{4S\ 12S}{12}$ $2S\ 3$ $\frac{11S}{12}$

12.(C) $\lim_{n\to\infty}6\tan\sum_{r=1}^{n}\tan^{-1}\frac{1}{1\ r\ 1\ r\ 2}$

Ÿ $\lim_{n\to\infty}6\tan\sum_{r=1}^{n}\tan^{-1}r\ 2$ $\tan^{-1}r\ 1$

Ÿ $\lim_{n\to\infty}6\tan\tan^{-1}n\ 2$ $\tan^{-1}2$

Ÿ $\lim_{n\to\infty}6\tan\tan^{-1}\frac{n}{1\ 2n\ 2}$

Ÿ $\lim_{n\to\infty}6\frac{n}{2n\ 5}$

Ÿ $\lim_{n\to\infty}\frac{3}{1\ \frac{5}{n}}$ 3

13 .(29) $50\tan\left(3\tan^{-1}\frac{1}{2}+2\cos^{-1}\frac{4}{\sqrt{5}}+\tan^{-1}\frac{1}{2}\right)=2^{1}2\sqrt{}$

$$50\tan\left(3\tan^{-1}\frac{1}{2}+2\cot^{-1}\frac{2}{1}+\frac{4}{2\sqrt{}}\right)$$

$$50\tan\left(S\tan^{-1}\frac{1}{2}+\frac{4}{2}\right)$$

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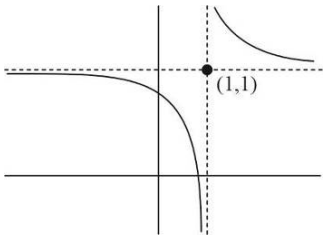
1.(D) $f(x) \quad 2x \quad 1$

$g(x) \quad \frac{x \quad 1/2}{x \quad 1}$

? $f(g(x)) \quad 2g(x) \quad 1$

$$2\left(\frac{x-\frac{1}{2}}{x-1}\right)-1 \quad \frac{x}{x-1} \quad \frac{x-1}{x-1} \quad 1 \quad \frac{1}{x-1}$$



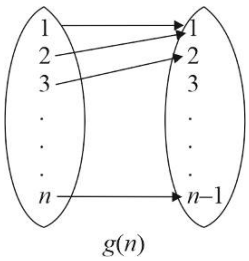
? $f(g(x))$ is oneone. But not onto.

2.(2) $a\left(f\left(\frac{1}{x}\right)+D\right)+b\left(x\left(\frac{1}{x}\right)\right)=\frac{E}{x}$

$af\left(\frac{1}{x}\right)+D(x)=\frac{b}{x}+xE$

Add $(a \quad D)f(x)+(a \quad D)f\left(\frac{1}{x}\right)+b\left(x+\frac{1}{x}\right)+xE+\frac{1}{x}$ Ÿ $f(x)+f\left(\frac{1}{x}\right)+2\left(x+\frac{1}{x}\right)$

3.(B)



For the function $g(n)$ as shown above g is onto but fog is not one -one

4.(D) $f(x) \quad \frac{5^x}{5^x \quad 5}$

$f(2 \quad x) \quad \frac{5}{5^{2-x} \quad 5} \quad \frac{5}{5 \quad 5^x}$ Ÿ $f(x) \quad f(2 \quad x) \quad 1.$

? Sum = 19 + $f(1) = 19 + \frac{5}{10} \quad 19 \quad \frac{1}{2} \quad \frac{39}{2}$

5.(1) $\log_4(x \quad 1) \quad \log_2(x \quad 3)$

Here $x \quad 1 \; ! \; 0, x \quad 3 \; ! \; 0$

$x \; ! \; 1, x \; ! \; 3$

$x \; ! \; 3$

Now $\log_4(x \quad 1) \quad \log_2(x \quad 3)$

$\frac{1}{2}\log_2(x \quad 1) \quad \log_2(x \quad 3)$

$\log_2 x \quad 1 \quad \log_2 x \quad 3^{\ 2}$

$x \quad 3^{\ 2} \quad x \quad 1$

$x^2 \quad 7x \quad 10 \quad 0$

$x \quad 2, 5$

$x \quad 5$ (Possible)

Hence only one solution

6.(C) $f \quad x \quad \sin^{\ 1} x \quad g \quad x \quad \dfrac{x^2 \quad x \quad 2}{2x^2 \quad x \quad 6}$

$\dfrac{x \quad 2 \quad x \quad 1}{x \quad 2 \quad 2x \quad 3} \quad \dfrac{x \quad 1}{2x \quad 3} \qquad x \ z \ 3/2$

$x \quad 2$ is taken as $\lim\limits_{x \ o \ 2} g \quad x \quad g \ 2 \quad \dfrac{3}{7}$

$fog \quad \sin^{\ 1} \quad g \quad x$

$1 \ dg \quad x \quad d1$

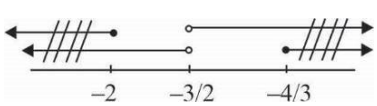
$\dfrac{x \quad 1}{2x \quad 3} \ d1 \ ; \ \dfrac{x \quad 1 \quad 2x \quad 3}{2x \quad 3} \ d0 \ ; \ \dfrac{x \quad 2}{2x \quad 3} \ t \ 0$

$(\quad f, \quad 2] \ ‰ \ \dfrac{§ \ 3}{©2}, \quad f \qquad \ldots(i)$

and $\dfrac{x \quad 1}{2x \quad 3} \ t \quad 1 \ ; \ \dfrac{x \quad 1 \quad 2x \quad 3}{2x \quad 3} \ t \ 0 \ ; \ \dfrac{3x \quad 4}{2x \quad 3} \ t \ 0$

$\dfrac{§}{©} \ f, \dfrac{3}{2} \ ‰ \ \dfrac{a4}{3}, \quad f \qquad \ldots(2)$

$1 \ \hat{} \ 2$



$(\quad f, \quad 2] \ ‰ \ \dfrac{a}{\ \ 3}, \quad f$

7.(B) Domain of f is $[0, \ f)$ and g is $(\quad f, 1]$

Domain of $f + g = [0, 1]$ and $f - g$ is $[0, 1]$

f/g is $[0, 1)$ and g/f is $(0, 1]$

Common domain $(0, 1)$

8.(D) $y \quad 5^{\log x}$ Ÿ Express x in terms of y

9.(D) $f(x) \quad \dfrac{x \quad 2}{x \quad 3}$ Ÿ $f^{\ 1}(x) \quad 3 \quad \dfrac{1}{x \quad 1} \ ; \ g(x) \quad 2x \quad 3$ Ÿ $g^{\ 1}(x) \quad \dfrac{x \quad 3}{2}$

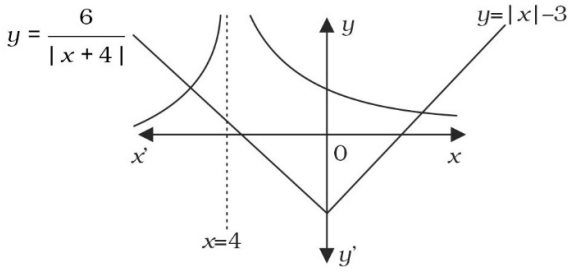
$f^{\ 1}(x) \quad g^{\ 1}(x) \quad 3 \quad \dfrac{1}{x \quad 1} \quad \dfrac{x \quad 3}{2} \quad \dfrac{13}{2} \quad \dfrac{x \quad 3}{2} \quad \dfrac{1}{x \quad 1} \quad \dfrac{13}{2} \quad 3 \quad \dfrac{7}{2}$

$\dfrac{(x \quad 3)(x \quad 1) \quad 2}{2(x \quad 1)} \quad \dfrac{7}{2} \quad x^2 \quad 2x \quad 3 \quad 2 \quad 7x \quad 7 \quad x^2 \quad 5x \quad 6 \quad 0$

$x \quad 2$ or 3 Sum = 5.

10 .(C) For x z 4

$|x| - 3 \quad \frac{6}{|x-4|}$



? No. of solutions = 2

11 .(A) Claim : f is one -one

Proof : say h(x) g(f (x))

If f is not one -one, say for $x_1 \neq x_2$ $f(x_1) = f(x_2)$

Ÿ $h(x_1) = h(x_2)$. So h is NOT one -one.

Claim: g is ONTO

Proof : f : A o B and g: B o C

gof : A o C

If g is NOT onto then gof will NOT take all values from C.

12 .(A) g : N o N

g 4 5, g 5 6, g 6 4

g 7 8, g 8 9, g 9 7

g(x) is onto

onto (onto) = onto if f x is onto then f g x onto

13 .(D) $e^{6x} + e^{4x} + 2e^{3x} + 12e^{2x} + e^{x} + 1 = 0$

$e^{6x} + 2e^{3x} + 1 + e^{4x} + e^{x} + 12e^{2x} = 0$

$(e^{3x} + 1)^2 + e^{x}(e^{3x} + 1) + 12e^{2x} = 0$

$(e^{3x} + 1) + 4e^{x} \cdot (e^{3x} + 1) + 3e^{x} = 0$

Now $e^{3x} + 4e^{x} + 1 = 0$ or $e^{3x} + 3e^{x} + 1 = 0$

$e^{x} = y$ Ÿ y m 0, f

$f(y) = y^3 + 4y + 1$; $f'(y) = 3y^2 + 4 > 0$

$y = \frac{2}{\sqrt{3}}$ Ÿ $x = \log_e\left(\frac{2}{\sqrt{3}}\right)$

Only one real root

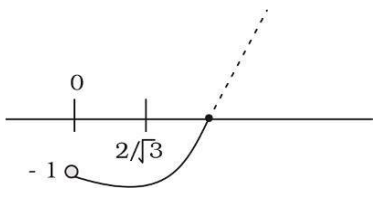
And $g(y) = y^2 + 3y + 1$

$g'(y) = 3y^2 + 3 > 0$

g 0 1

g 1 3

Only one real root So total two real roots



14.(C) For domain

$\frac{|x|\ 2}{|x|@3}$ t 0

Case I: when $|x|$ 2 t 0

And $|x|$ 3 ! 0

Ÿ x • f , 3 ‰ 4, f …(i)

Case II: When $|x|$ 2 d0

And $|x|$ 3 0

Ÿ x • 2,3 …(ii)

So from (i) and (ii)

x • f , 3 ‰ 2,3 ‰4, f

? a b c 3 2 3 2

15 .(A) f f (x) $\frac{5\ 5x\ 3}{6x\ D}$ 3 Y $\frac{25x\ 15\ 18x\ 3\ D}{30x\ 18\ D6x\ D}$ $\frac{43x\ 15\ 3\ D}{x\ 30\ 6\ D\ 18\ D^2}$ Ÿ 15 3 D 0 ; D 5

6 $\frac{5x\ 3}{6x\ D}$; D

16 .(B) f x x

18 6 Ÿ O 3

f x 3x

f 2 . f 3 6.9 54

17 .(256) A f,1 ‰3, f

B f, 2 ‰2, f

C f,2 ¼‰6, f ? A ˆ B ˆ C f , 2 ‰ 6, f Ÿ A ˆ B ˆ C ' ¬ 2,6

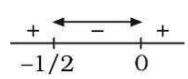
? A ˆ B ˆ C 'ˆ Z ' 2, 1,0,1,2,3,4,5 `

Hence total no. of subsets = 2^8 256

18 .(D) $\frac{1\ x}{x}$ t 1 or $\frac{1\ x}{x}$ d 1

Ÿ $\frac{1}{x}$ t 0 $\frac{1\ x}{x}$ 1 d0 Ÿ $\frac{1\ 2x}{x}$ d0

Ÿ x ! 0 ……(i)



Ÿ x • ¬ $\frac{1}{2}$, 0 ¸ ……(ii)

Taking union of (i) and (2)

x • ¬ $\frac{1}{2}$, f ¸ {0}

19 .(B) f (x) $\log_{\sqrt{5}}[3\ \sqrt{2}\ (\cos x\ \sin x]$

? y • [0, 2]

Solutions ofArchive - JEE Main & Advanced

Functions	Class- XII | Mathematics

JEE Main 2022

1.(1440)

A : – x 1 x 9 d0

Ÿ x • 1,9 ÿ x • 1,2,3,4,5,6,7,8,9

f x d x 3 2 1 Ÿf 1 d5 o1 2,2 2 o2 possibility

f 2 d2 o1 2 o1 possibility

f 3 d1 o1 2 o1 possibility

f 4 d2 o1 2 o1 possibility

f 5 d5 o1 2,2 2 o2 possibility

f 6 d10 o1 2,2 2,3 2 o3 possibility

f 7 d17 o1 2,2 2,3 2,4 2 o4 possibility

f 8 d26 o1 2,2 2,3 2,4 2,5 2 o5 possibi lity

f 9 d37 o1 2,2 2,3 2,4 2,5 2,6 2 o6 possibility

Ÿ total 2 u6 1440

2.(C) 1 d2x 2 3 2

2 d2x 2 5

1 dx 2 $\frac{5}{2}$

$\sqrt{\frac{5}{2}}$ x d 1 or 1 dx $\sqrt{\frac{5}{2}}$

$x^2 - 5x + 5 < 0$

and $\log_{\frac{1}{2}}(x^2 - 5x + 5) < 0$

Ÿ $x^2 - 5x + 5 < 0$ and $x^2 - 5x + 5 > 1$

Ÿ $x \in \left(\frac{5-\sqrt{5}}{2}, 1\right) \cup \left(4, \frac{5+\sqrt{5}}{2}\right)$, f and $x \in (1,4)$

Combining all $x \in \left(1, \frac{5+\sqrt{5}}{2}\right)$

3.(112) $n(C \cap B)$ can be 1, 2, 3

Also $C \cap B$ cannot contain the element '9' as $C \subset A$, and $9 \in A$

Case-1 $n(C \cap B) = 1$ $A = \{1,2,3,4,5,6,7\}$

selecting 1 out of 3, 6, 7 and remaining 4 elements each has 2 options will belong to C or will not belong to C.

Number of set $C = {}^3C_1 \times 2 \times 2 \times 2 \times 2 = 48$

Case 2 $n(C \cap B) = 2$

selecting 2 out of 3, 6, 7 and remaining 4 elements each has 2 options will belong to C or will not belong to C.

Number of set $C = {}^3C_2 \times 2 \times 2 \times 2 \times 2 = 48$

Case 3 $n(C \cap B) = 3$

Selecting all 3 of 3, 6, 7 and remaining 4 elements each has 2 options will belong to C or will not belong to C.

Number of set $C = {}^3C_3 \times 2 \times 2 \times 2 \times 2 = 16$

Total = 48 + 48 + 16 = 112

4.(11) $S = \{4,6,9\}$ $T = \{9,10,11.....1000\}$

$A = \{a_1, a_2, ...a_k : K \in N\}$ and $a_i \in S$

Here by the definition of set 'A'

$A = \{a : a = 4x + 6y + 9z\}$

Except the element 11, every element of set T is of the form $4x + 6y + 9z$ for some $x, y, z \in W$

$\therefore T \subset A = \{11\}$

5.(B) Number of possible values of $a = 60$, for $b = pq$,

If $p = 3$, $q = 3, 5, 7, 11, 13, 17, 19$

If $p = 5$ $q = 5, 7, 11$

If $p = 7$ $q = 7$

Total cases = 60 × 11 = 660

6.(C) $1 \le \frac{x^2 + 3x + 2}{x^2 + 2x + 7} \le 1$

$\therefore x \in \frac{a}{\neg}1, f$

7.(A) $2\cos\left(\frac{x^2 + x}{6}\right) = 4^x + 4^{-x}$

L.H.S. ≥ 2 & R.H.S ≤ 2

$\therefore 2\cos\left(\frac{x^2 + x}{6}\right) = 2$

$x = 0$

8.(D) Using A.M, G.M. inequality

i.e., $AM \ge GM$, we get that

$\frac{x + x + x + y + y}{5} \ge (x \cdot x \cdot x \cdot y \cdot y)^{1/5}$

$\frac{3x + 2y}{5} \ge (x^3 y^2)^{1/5}$

$\frac{3x + 2y}{5} \ge (2^{15})^{1/5}$

$\therefore (3x + 2y)_{min} = 40$

9.(C) $f(2) \quad \lim_{h \to 0} f(2 \quad h) \quad \lim_{h \to 0} \frac{\sin h}{h} \quad 1$

? f is conti nuous at $x \quad 2$

$f(1) \quad \lim_{h \to 0} \frac{\sin(1 \quad h \quad 2)}{(1 \quad h \quad 2)} \quad \sin 1$

$f(1) \quad 0, \; f(1) \quad 0$

$f(1) \quad 1 \; \& \; f(1) \quad 0$

? f is not continuous at $x \quad 1$

? f is discontinuous at $x \quad 1,1 \quad ?m \quad 2$

f is not diff. at $x \quad 1,0 \; \& \; 1 \quad ? \; n \quad 3$

10.(100) $A \quad \begin{matrix} a & 1 & a \\ « & & \\ 0 & & b \end{matrix}$

$A^2 \quad \begin{matrix} a & 1 & a \\ « & & \\ 0 & & b \end{matrix} \quad \begin{matrix} o & a & 1 & a \\ » & & « & \\ 1/4 & & 0 & b \end{matrix}$

$\begin{matrix} a & 1 & a & ab \\ « & & & \\ 0 & & b^2 & \end{matrix}$

? $T_n \quad \{A \bullet S; A^{n(n \quad 1)} \quad I\}$ Here, $n(n \quad 1)$ is always even

? b must be equal to 1

? $T_1, T_2, T_3, \ldots, T_n$ are all I for $b \quad 1$ and a can take any value from 1 to 100

$\begin{matrix} 100 \\ \\ n \quad 1 \end{matrix}$

? $T_n \quad 100$

11.(D) $1 \; d \frac{x^2 \quad 5x \quad 6}{x^2 \quad 9} \quad d1 \quad \ddot{Y}x \bullet \begin{matrix} a \\ « \\ « \end{matrix} \frac{1}{2}, \quad f$

$x^2 \quad 3x \quad 2 \; !0 \quad \ddot{Y} \; x \bullet (\quad f,1) \; \%(2, \quad)f$

And $x \quad z \frac{3 \quad r\sqrt{5}}{2}$

12.(31) $3 \; f(c) \quad 2 \; f(a) \quad f(d) \quad f(b)$

3(0) 2 1 2 4
3 5
4 6
5 7
6 8
7 9
8 10

3(0) 2 2 1 5
3 7
4 8
5 9
6 10

3(0) 2 3 1 7
2 8
4 10

3(0) 2(4) 1 9
2 10

3(1) 2 (0) 2 5
3 6
4 7
5 8
6 9
7 10

3(1) 2 (2) 0 7
3 10

3(1) 2 (3) 0 9

3(2) 2(0) 1 7
3 9
4 10

3(2) 2(1) 0 8
3(3) 2(0) 1 10

Total 31 ways.

13 .(34) Let $y = \frac{9 - x^2}{5 - x} - \frac{25 - x^2}{5 - x} - \frac{16}{5 - x} - (5 - x) - \frac{16}{5 - x}$

$\frac{dy}{dx} = 1 - \frac{16}{(5-x)^2}$, for $\frac{dy}{dx} = 0 \Rightarrow 16 = (5-x)^2 \Rightarrow x = 1$ [illegible] $0 \le x \le 2$

? Critical point is 1

$y(0) = \frac{9}{5}, y(1) = 2, y(2) = \frac{5}{3}$

So, $D = 2$ & $E = \frac{5}{3}$

$$3 \max\left\{\frac{9 - x^2}{5 - x}, x\right\} dx \quad \max\left\{\frac{9 - x^2}{5 - x}, x\right\} dx$$

$\frac{5}{3}$ $\frac{8}{3}$

$$3\frac{9 - x^2}{5 - x} dx \quad 3x dx$$

$$I \quad 35 - x - \frac{16}{5 - x} dx \quad x3 dx$$

$$I = 14 - \frac{28}{25} - 16\log\left(\frac{8}{15}\right) \cdot \frac{72}{25}$$

$D_1 = 18$ & $D_2 = 16$

14 .(B) (I) $|x - 1| \ge 2$
$\Rightarrow -2 \ge x - 1 \ge 2$
$\Rightarrow -3 \ge x \ge 1$
$\Rightarrow x \in (-\infty, -3] \cup [1, \infty)$
$A : x \in (-\infty, -3] \cup [1, \infty)$ ...(i)

(II) $|x - 1| \le 2$
$x - 1 \le 2$ or $x - 1 \ge -2$
$\Rightarrow x \le 3$ or $x \ge -1$
$\Rightarrow x \in (-\infty, \infty)$
$B : x \in (-\infty, \infty)$ ...(ii)

$B \cap A = R - (-3, 1)$

15 .(C) f x y 2f x f y

Given f 1 2

f 2 f 1 1 2 f 1 . f 1 2.2 ^{2} 2 ^{3}

f 3 f 2 1 2 f 2 f 1 2 2 ^{3} 2u 2 ^{5}

10 f D k 10 f D k 2f D f x
k 1 k 1 k 1

2 f D f 1 f 2 f 3 ..... f 10

2 f D 2 2 ^{3} 2 ^{5} ..... 2 f D 2 u 2 ^{20} 1 / 4 1

2 f D 2 2 ^{20} 1 / 3 ½ ¾ 512 / 3 2 ^{20} 1 ; 4 f D 512 Ÿf D 512 / 4 128

f D 128 Ÿ D 4

16 .(A) f x x ^{3} x 5

f g x x Ÿg is f ^{1} Ÿg f x x

g' f x .f ' x 1 Ÿg' f x 1 / f ' x

f ' x 3x ^{2} 1 Ÿg' f x 1 / 3x ^{2} 1

f 4 4 ^{3} 4 5

64 1 63

g' f 4 1 / 3 4 ^{2} 1 1 / 49

g' 63 1 / 49

17 .(A) f x log$_{e}$ x ^{2} 1 e ^{x} 1 & g x 1 2e ^{2x} / e ^{x}

f g D 1 ^{2} / 3 ! f g D 5 / 3

f ' x 2x / 1 x ^{2} e ^{x} ! 0 x • R

g x e ^{x} 2e ^{x} Ÿ g' x e ^{x} 2e ^{x} 0 x • R

f x is increasing & g x is decreasing function

f g D 1 ^{2} / 3 ! f g D 5 / 3

g D 1 ^{2} / 3 ! g D 5 / 3 ; D 1 ^{2} / 3 ! D 5 / 3

D 1 ^{2} 3 D5

D^{2} 1 2 D 3 D5

D^{2} 5 D 6 0

D 2 D3 0

D • 2,3

18 .(2) f x $\left\{ 2 \left(\frac{x^{25}}{2} \right) \right\}^{1/50}$

g x f f f x f f x

g 1 f f f 1 f f 1

f 1 $3^{1/50}$

f f 1 f $3^{1/50}$ $\left\{ 2 \left(\frac{3^{1/2}}{2} \right) \right\}^{1/50}$

$\left\{ \frac{2 \; 2 \; \sqrt{3} \; 2 \; \sqrt{3}}{2} \right\}^{1/50}$ 1

? g 1 f f f 1 f f 1 f 1 1 $3^{1/50}$ 1

g 1 $3^{1/50}$ 1 $3^{1/50}$ 1 1 1 2

19 .(D) Notice g is even. So, fog can't be one -one.

Notice for no value of x f (g (x)) 1. So f is NOT onto.

20 .(248) f (x y) 2^{x} f (y) 4^{y} f (x), f (2) 3

Swapping x and y

f (y x) 2^{y} f (x) 4^{x} f (y)

Ÿ 2^{x} f (y) 4^{y} f (x) 2^{y} f (x) 4^{x} f (y) Ÿ f (x) (4^{y} 2^{y}) f (y) (4^{x} 2^{x})

Ÿ $\frac{f(x)}{4^{x} \; 2^{x}}$ $\frac{f(y)}{4^{y} \; 2^{y}}$ k (constant)

Ÿ f (x) k (4^{x} 2^{x}) Ÿ f (x) k (4^{x} ln 4 2^{x} ln 2)

f (2x) k (4^{2x} ln 4 2^{2x} ln 2)

Ÿ $\frac{f(4)}{f(2)}$ $\frac{4^{4} \ln 4 \; 2^{4} \ln 2}{4^{2} \ln 4 \; 2^{2} \ln 2}$

$\frac{\ln 2(2^{9} \; 2^{4})}{\ln 2(2^{5} \; 2^{2})}$ $\frac{496}{28}$ $\frac{124}{7}$

14 $\frac{f(4)}{f(2)}$ 14 u $\frac{124}{7}$ 248